EE 435

Lecture 39

References and Bias Generators

Final Exam:

Scheduled on Final Exam Schedule:

Wednesday May 5 9:45 a.m.

Revised Final Exam:

- Take-home format – open book and open notes
- Will be posted on course WEB site by late Friday April 30
- Due at 5:00 p.m. on Wednesday May 5 : Upload as pdf file into Canvas

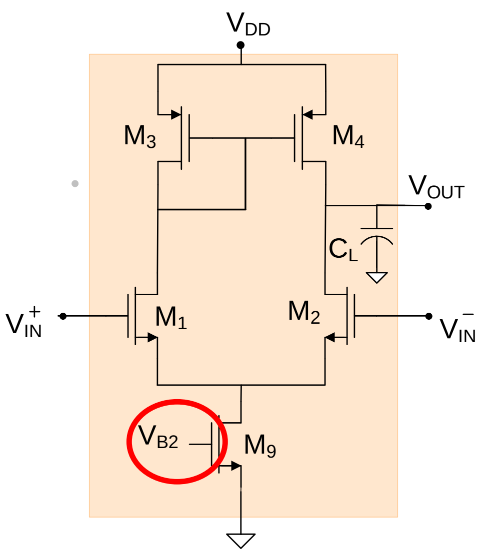
If anyone has any constraints of any form such as internet access or other factors that makes it difficult to work with this revised format, please contact Professor Geiger by 5:00 p.m. on Wednesday April 28

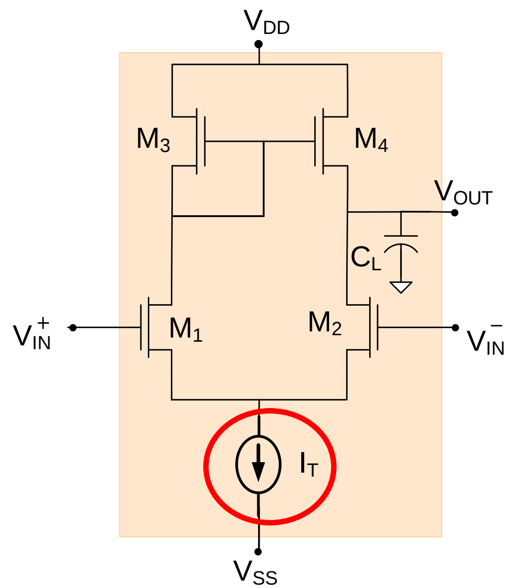
Bias Voltages/Currents and References

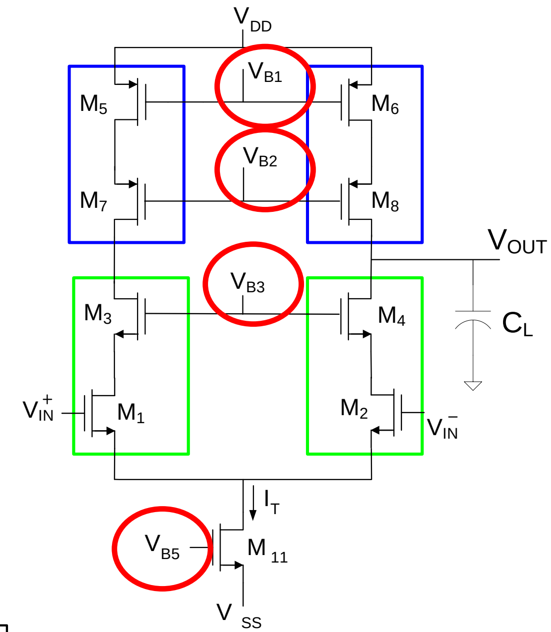
How do we get quantities such as voltage, current, resistance, temperature, ?.... in an electronic circuit

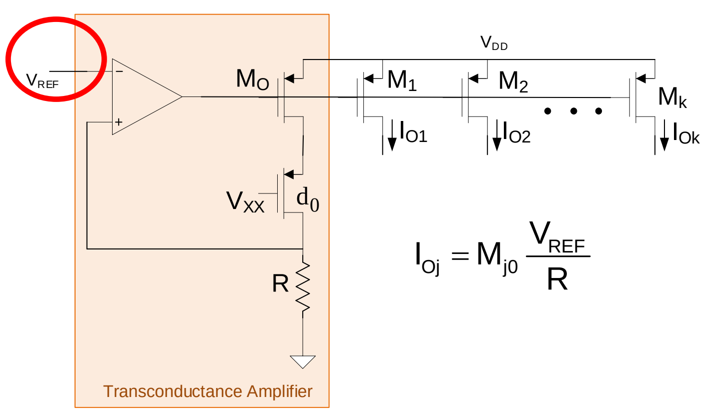
Bias Voltages/Currents and References

How are these voltages and currents generated?

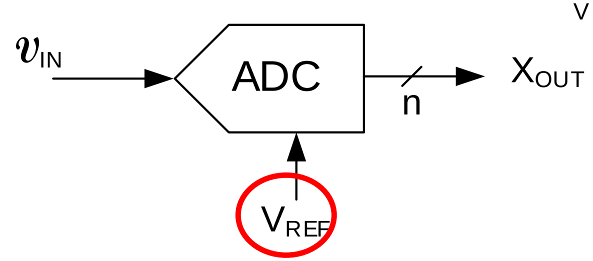


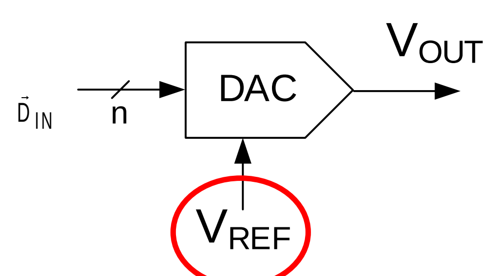






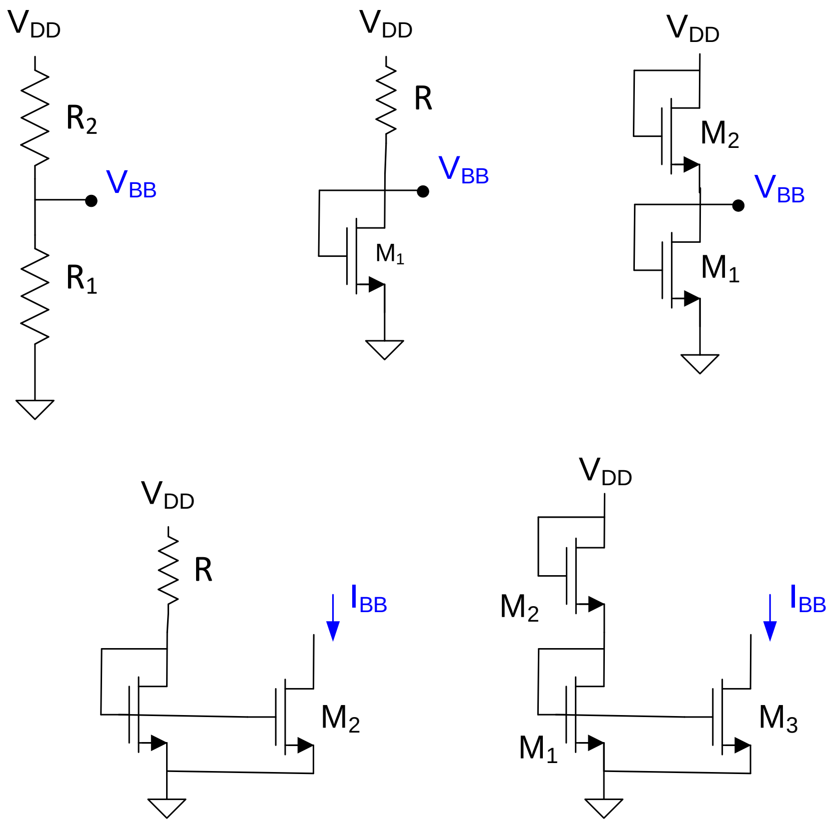
$$I_{Oj} = M_{j0}\frac{V_{REF}}{R}$$





Bias Voltages/Currents and References

How are these voltages and currents generated?



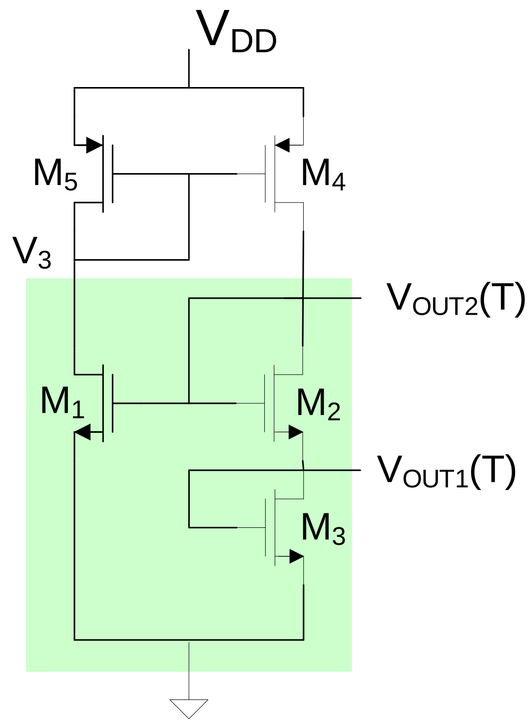
All will work !

Termed Supply-Referenced Sources

But supply sensitivity (supplies usually poorly controlled and noisy) process dependence, and temperature dependence unacceptable in many applications

Bias Voltages/Currents Generators

How are these voltages and currents generated?



Inverse-Widlar

$$V_{01} = V_{Tn}\left(\frac{1-\sqrt{\frac{M_{54}W_2L_1}{W_1L_2}}}{1+\sqrt{\frac{W_2L_3}{W_3L_2}}-\sqrt{\frac{M_{54}W_2L_1}{W_1L_2}}}\right)$$

$$V_{02} = V_{Tn}\left(\frac{1+\sqrt{\frac{W_2L_3}{W_3L_2}}-2\sqrt{\frac{M_{54}W_2L_1}{W_1L_2}}}{1+\sqrt{\frac{W_2L_3}{W_3L_2}}-\sqrt{\frac{M_{54}W_2L_1}{W_1L_2}}}\right)$$

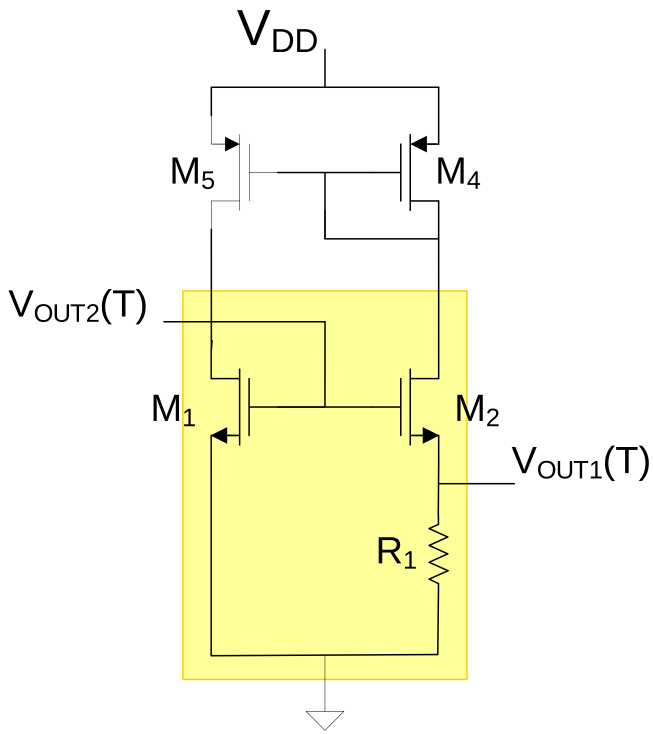
M_{54} is the M_5:M_4 Current Mirror Gain

Supply-independent Bias Generator!

Start-up circuit needed (notice positive feedback loop)

Supply-independent Bias Generators Widely Used

Bias Voltages/Currents Generators



Widlar Generator !

$$V_{01} = \left(\frac{\theta_1}{2} \pm \sqrt{\frac{\theta_1 V_{Tn}}{2} + \left(\frac{\theta_1}{2}\right)^2} \right) \left(1 - \sqrt{\frac{W_1 L_2}{M_{45} W_2 L_1}} \right)$$

$$V_{02} = V_{Tn} + \frac{\theta_1}{2} \pm \sqrt{\frac{\theta_1 V_{Tn}}{2} + \left(\frac{\theta_1}{2}\right)^2}$$

where

$$\theta_1 = \frac{M_{45} 2 L_1}{R \mu_n C_{OX} W_1}$$

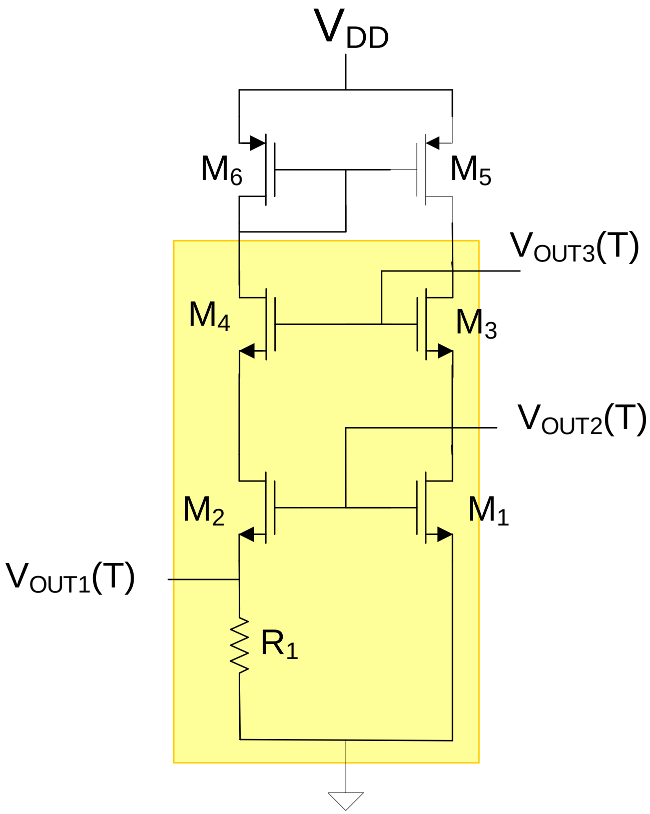
M_{45} is the M_4:M_5 Current Mirror Gain

Supply-independent Bias Generator!

Start-up circuit needed (notice positive feedback loop)

Supply-independent Bias Generators Widely Used

Bias Voltages/Currents Generators



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$$V_{01} = \left(\frac{\theta_1}{2} \pm \sqrt{\frac{\theta_1 V_{Tn}}{2} + \left(\frac{\theta_1}{2}\right)^2}\right)\left(1 - \sqrt{\frac{W_1 L_2}{M_{65} W_2 L_1}}\right)$$

$$V_{02} = V_{Tn} + \frac{\theta_1}{2} \pm \sqrt{\frac{\theta_1 V_{Tn}}{2} + \left(\frac{\theta_1}{2}\right)^2}$$

where

$$\theta_1 = \frac{M_{65} 2 L_1}{R \mu_n C_{OX} W_1}$$

M_{65} is the M_6:M_5 Current Mirror Gain

Widlar Generator !

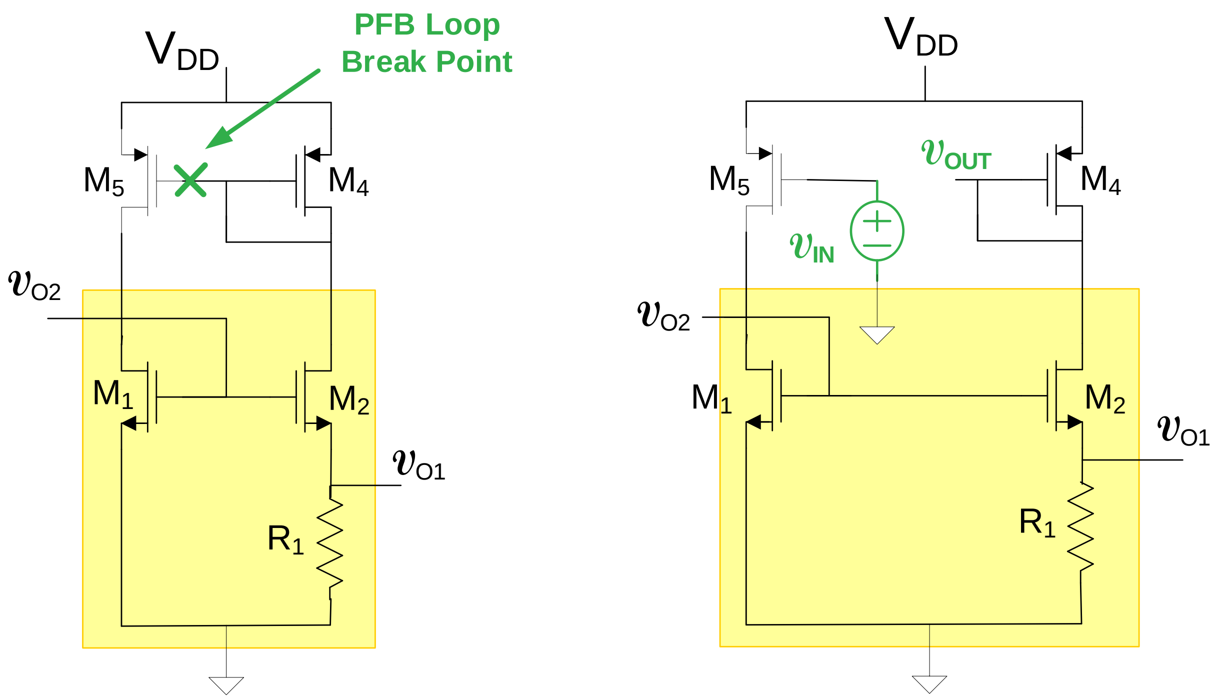
Supply-independent Bias Generator!

Start-up circuit needed (notice positive feedback loop)

Supply-independent Bias Generators Widely Used

Bias Voltages/Currents Generators

Need for Start-up Circuit



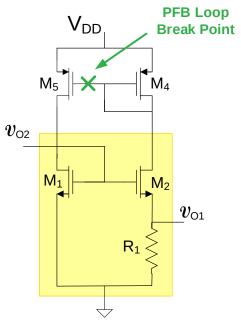
V_{OUT}=f(V_{IN}) termed the return map

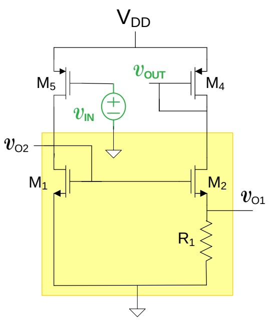
Termed Homotopy Analysis

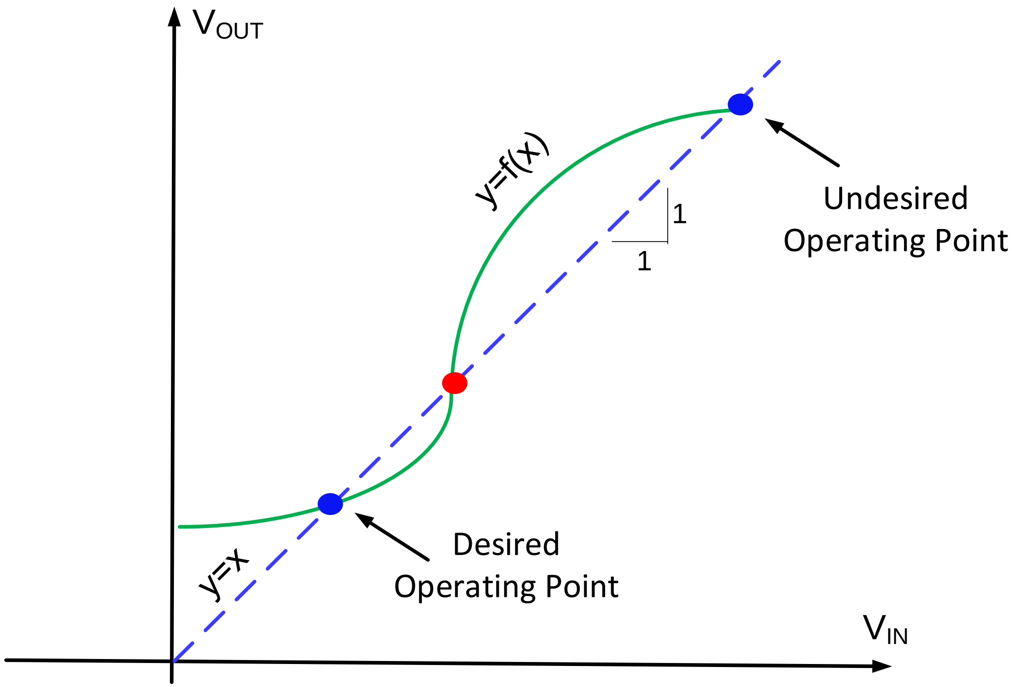
Must not perturb operating point when breaking loop !

Bias Voltages/Currents Generators

Need for Start-up Circuit

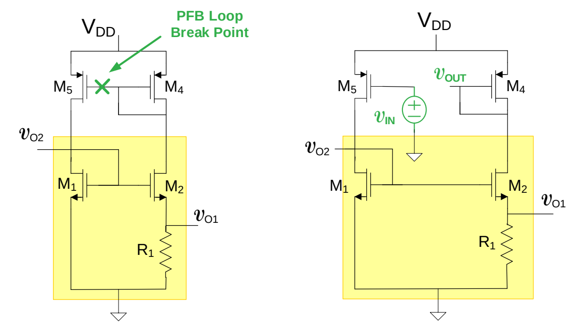


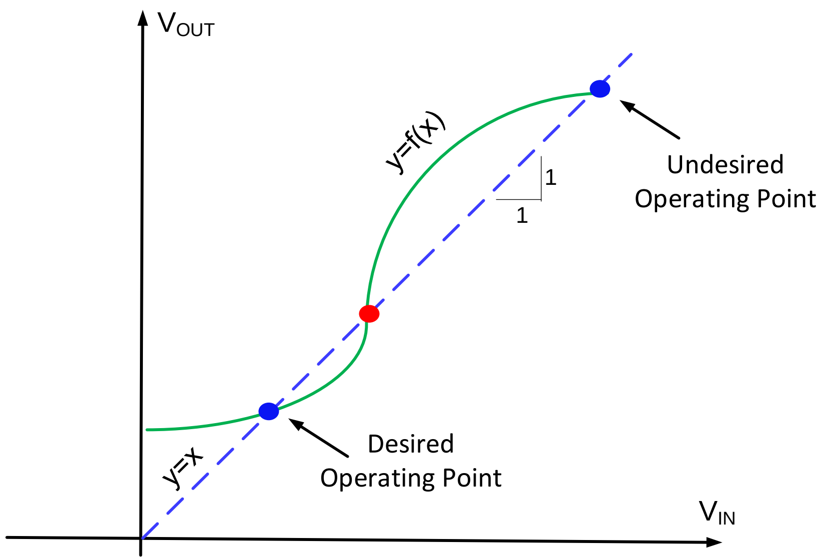




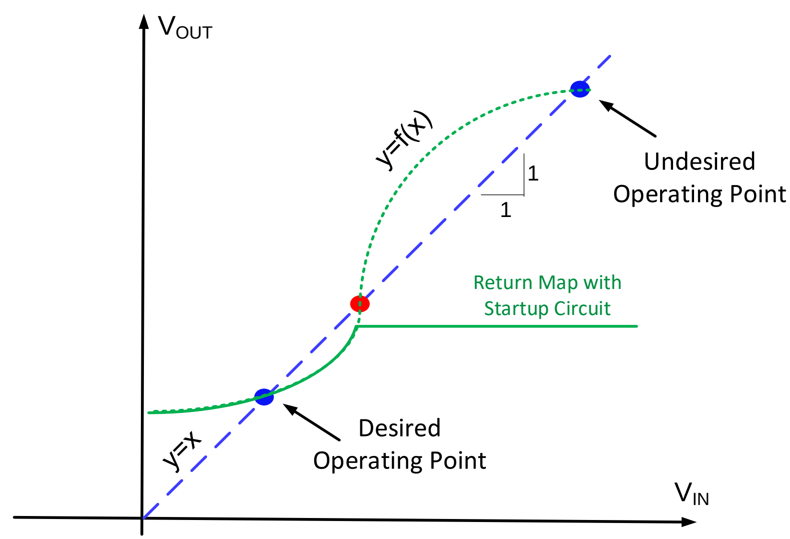
Bias Voltages/Currents Generators

Need for Start-up Circuit





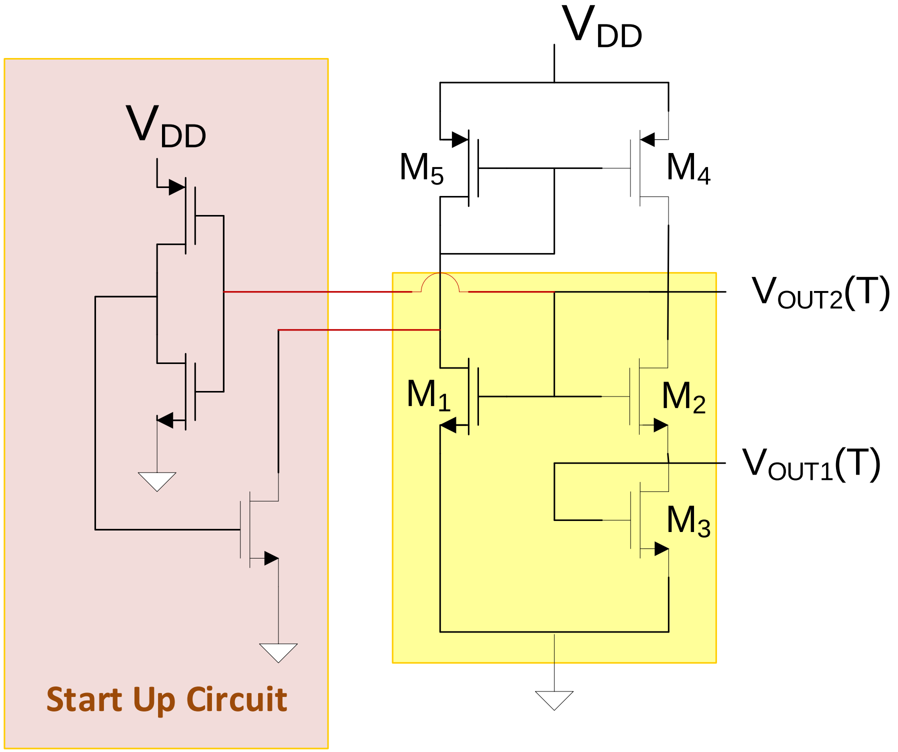
Without start-up circuit



With start-up circuit

Must verify start-up is effective over PVT variations

Bias Voltages/Currents Generators



Several different start-up circuits have been used

This start-up circuit shuts off during normal operation !

Bias Voltages/Currents Generators

Often prefer bias generators whose output changes with process parameters

Better biases many linear circuits (e.g. op amps)

But these bias generators, though simple, are process and temperature dependent

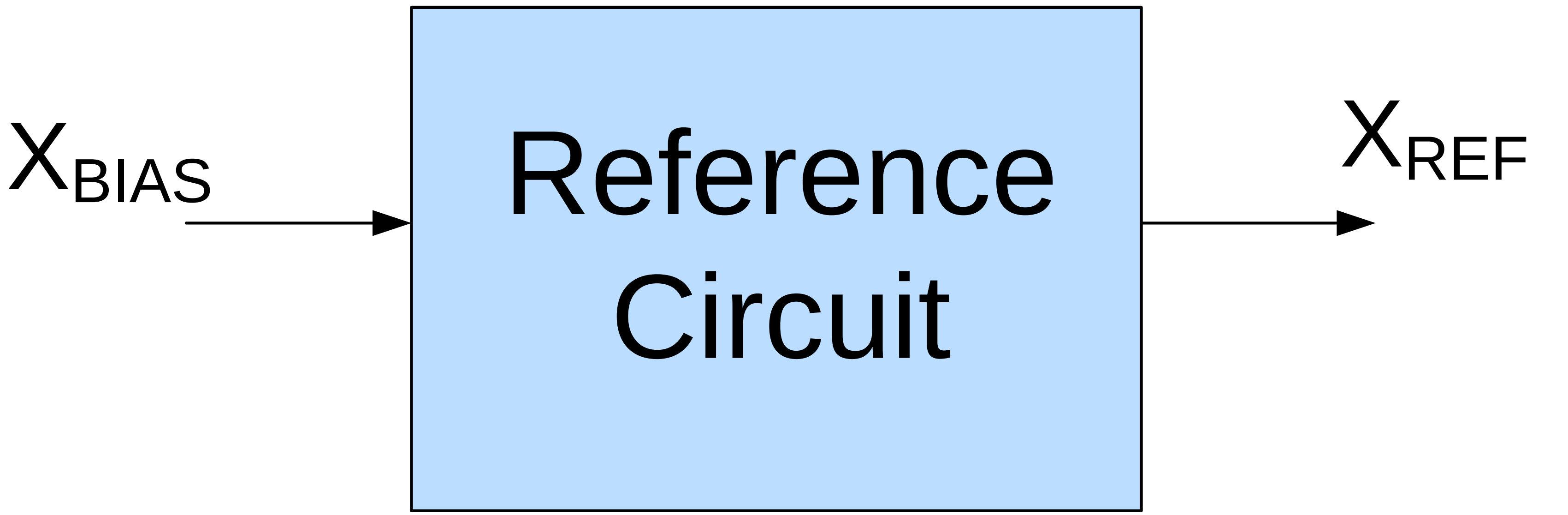
The term “References” usually refers to generators that are ideally independent of supply, process, and temperature

Types of References

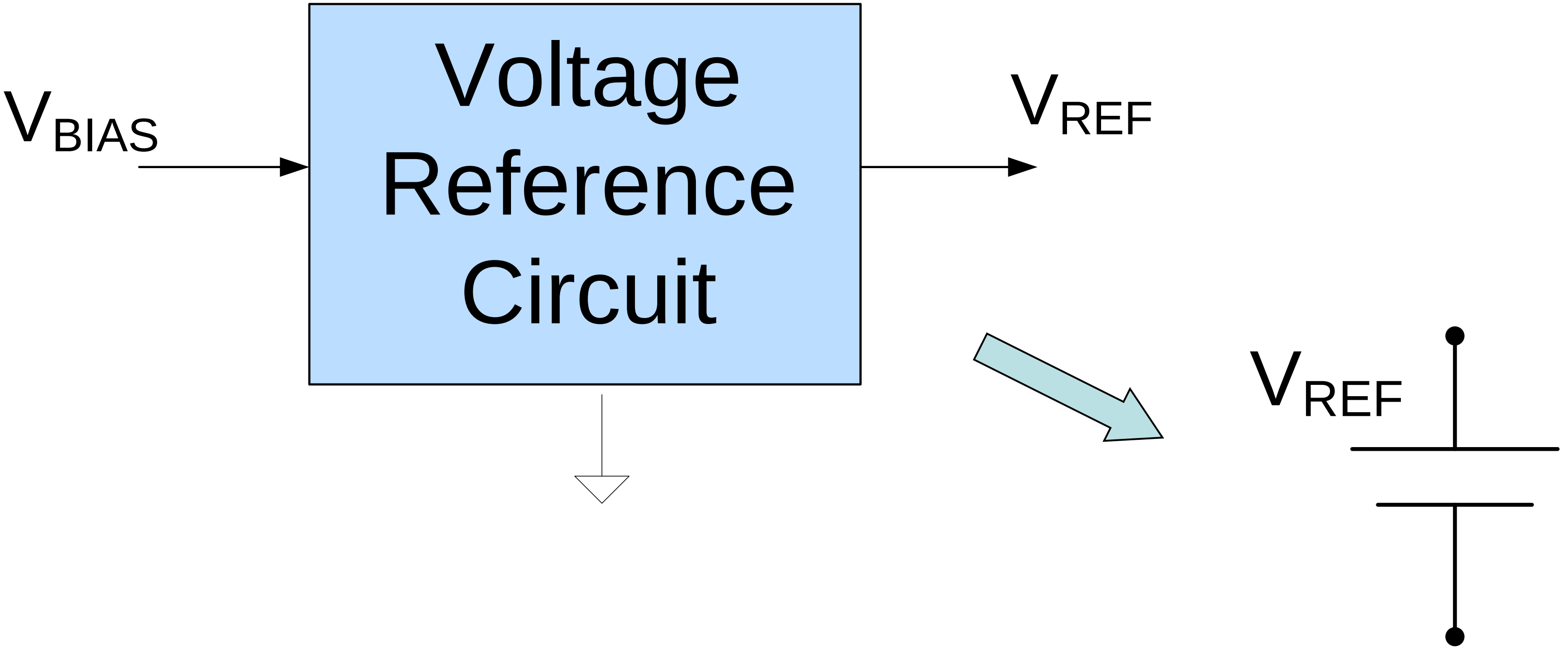
- Voltage References
- Current References
- Time References
- ….

Sensors Closely Related

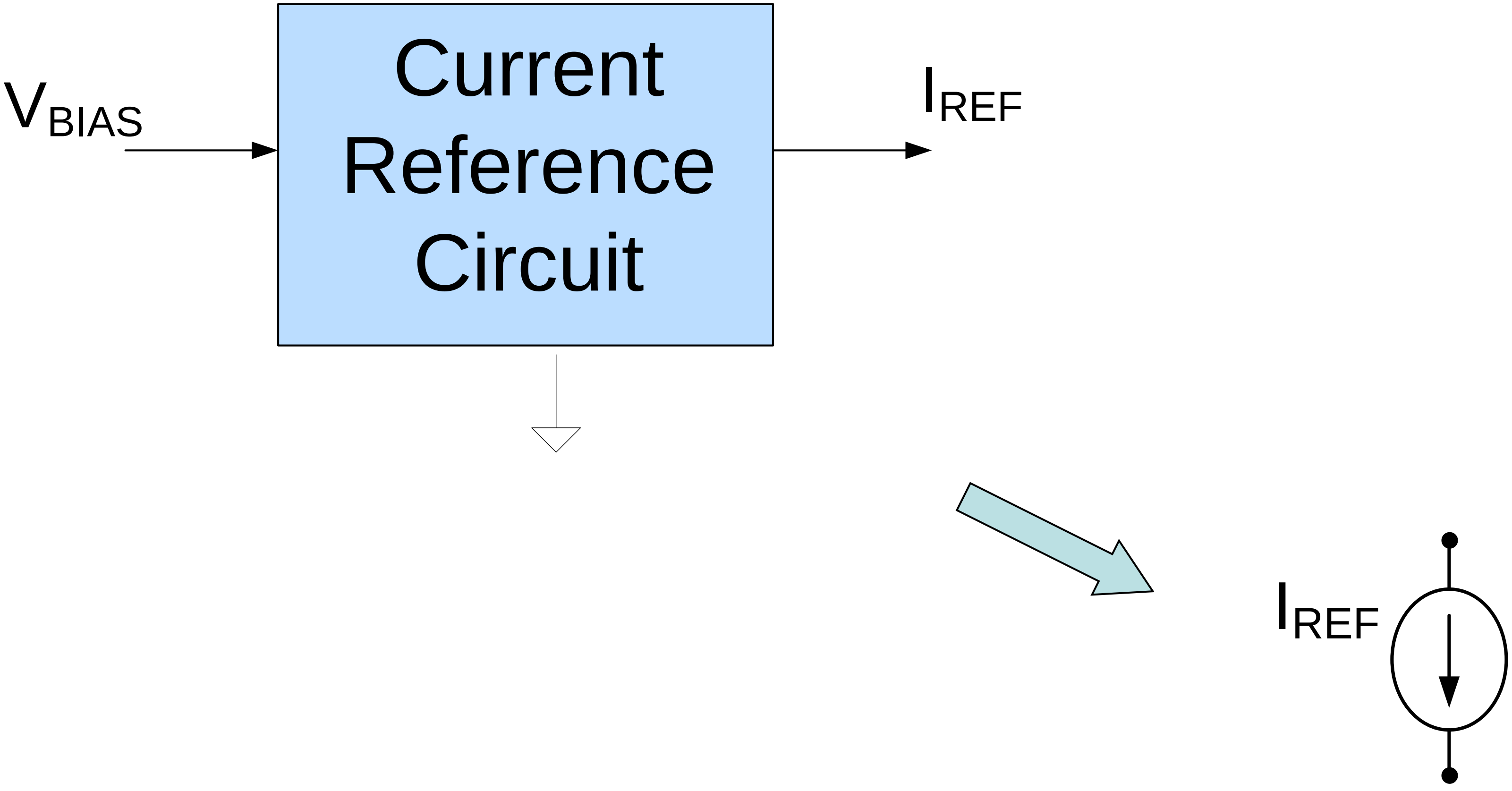
- Temperature
- Period
- Resistance
- Capacitance
- ….


X_BIAS
Reference
Circuit
X_REF

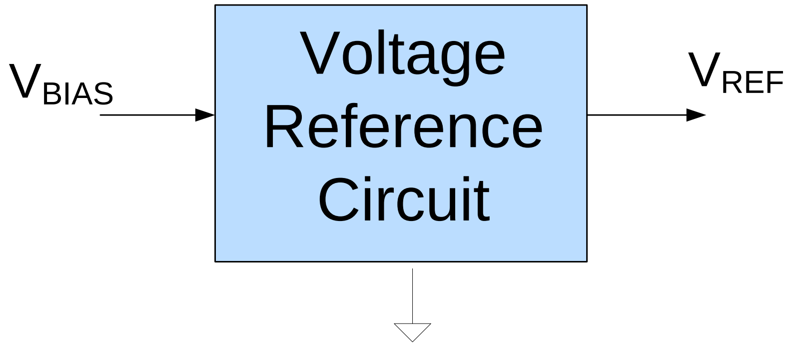
Voltage Reference



Current Reference



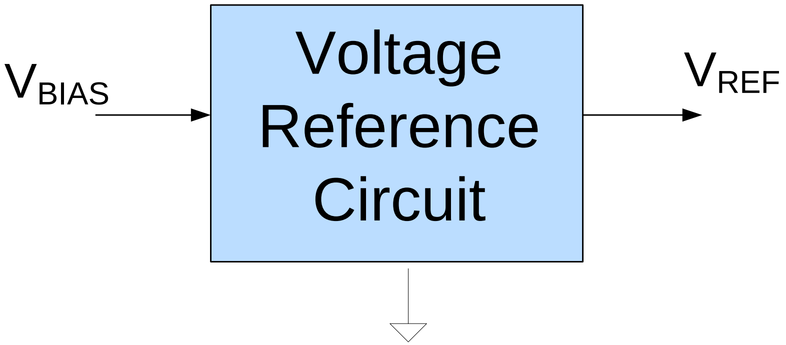
Desired Properties of References



- Accurate
- Temperature Stable
- Time Stable
- Insensitive to V_{BIAS}

- Low Output Impedance (voltage reference)
- Floating
- Small Area
- Low Power Dissipation
- Process Tolerant
- Process Transportable

Desired Properties of References

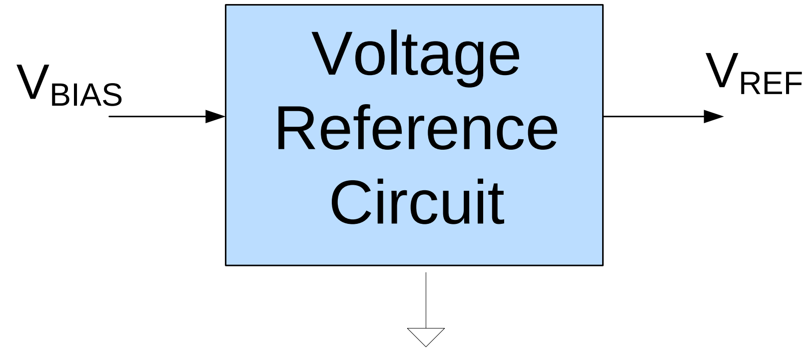


- Accurate √
- Temperature Stable √
- Time Stable √
- Insensitive to V_{BIAS} √

- Low Output Impedance (voltage reference)
- Floating
- Small Area
- Low Power Dissipation
- Process Tolerant √
- Process Transportable

Similar properties desired in other references

Consider Voltage References

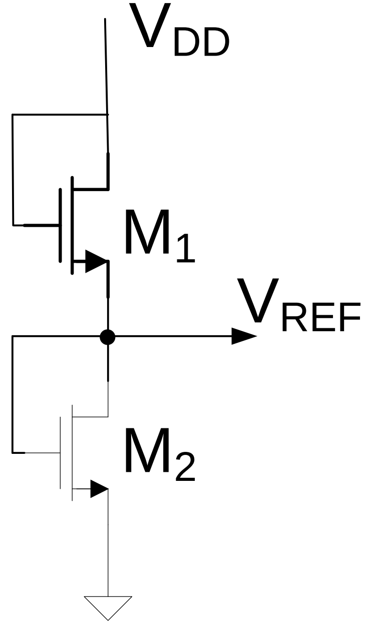


$$I_{D1} = \frac{\mu C_{OX} W_1}{2L_1}\left(V_{GS1} - V_{T1}\right)^2$$

$$I_{D2} = \frac{\mu C_{OX} W_2}{2L_2}\left(V_{GS2} - V_{T2}\right)^2$$

$$V_{T1} = V_{T0} + \gamma\left(\sqrt{\phi + V_{REF}} - \sqrt{\phi}\right)$$

$$V_{DD} - V_{REF} - V_{T1} = \sqrt{\frac{W_2 L_1}{W_1 L_2}}\left(V_{REF} - V_{T2}\right)$$

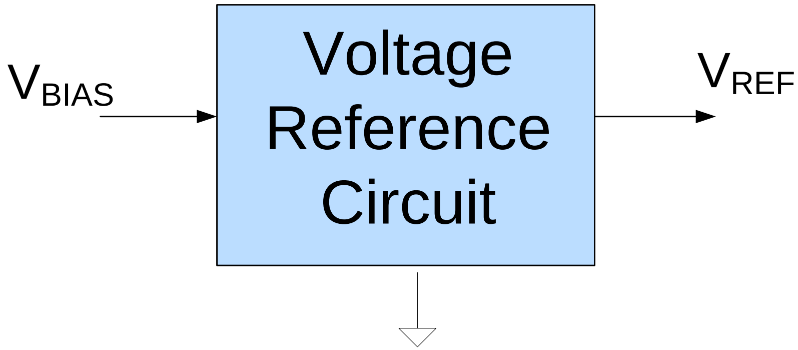


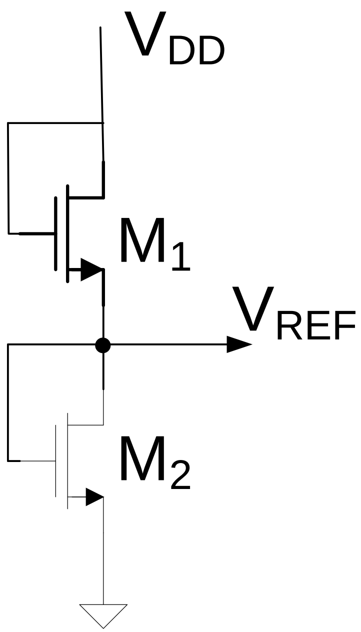
If matching assumed and γ effects neglected

$$V_{REF} = \frac{V_{DD} - V_{T0}\left(1 - \sqrt{\frac{W_2 L_1}{W_1 L_2}}\right)}{1 + \sqrt{\frac{W_2 L_1}{W_1 L_2}}}$$

Popular Voltage "Reference"

Consider Voltage References





Popular Voltage "Reference"

Uses as a reference limited to biasing and even for this may not be good enough !

If matching assumed and γ effects neglected

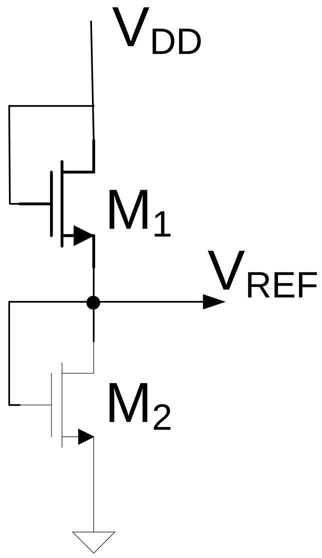
$$V_{REF} = \frac{V_{DD} - V_{T0}\left(1 - \sqrt{\frac{W_2 L_1}{W_1 L_2}}\right)}{1 + \sqrt{\frac{W_2 L_1}{W_1 L_2}}}$$

Dependent upon V_{DD}, V_{T0}, matching, process variations, γ

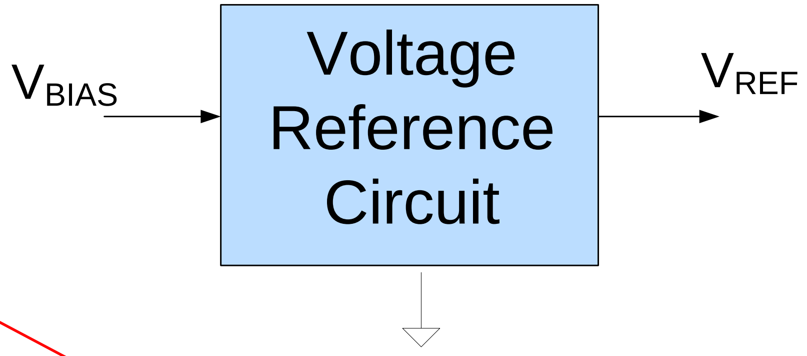
Termed a V_{DD},V_{TH} reference

Does not satisfy key properties of voltage references

Consider Voltage References



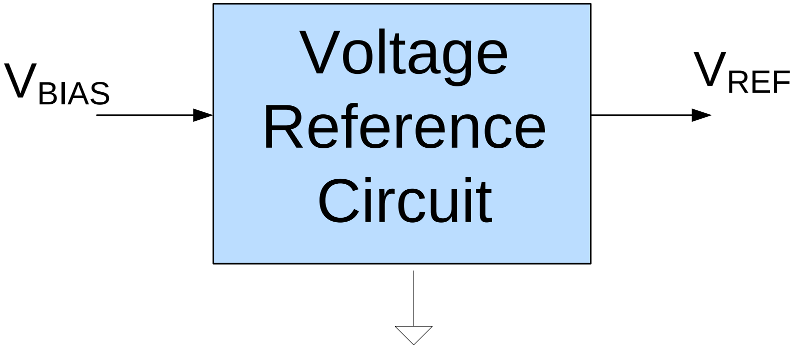
V_{DD},V_T reference



$$V_{REF} = \frac{V_{DD} - V_{T0}\left(1 - \sqrt{\frac{W_2 L_1}{W_1 L_2}}\right)}{1 + \sqrt{\frac{W_2 L_1}{W_1 L_2}}}$$

Observation – Variables with units Volts needed to build any voltage reference

Voltage References



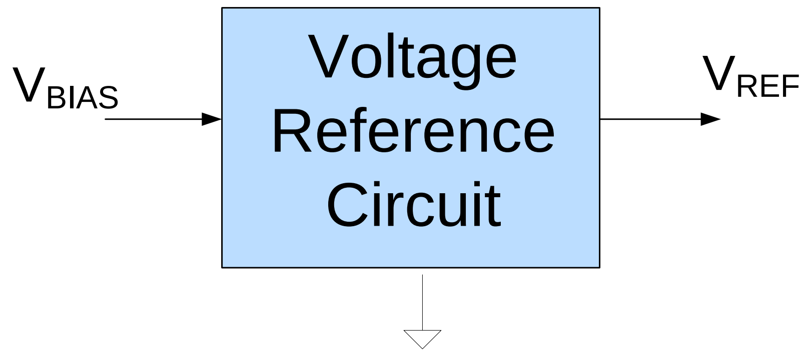
Observation – Variables with units Volts needed to build any voltage reference

What variables available in a process have units volts?

What variables which have units volts satisfy the desired properties of a voltage reference?

How can a circuit be designed that "expresses" the desired variables?

Voltage References



Observation – Variables with units Volts needed to build any voltage reference

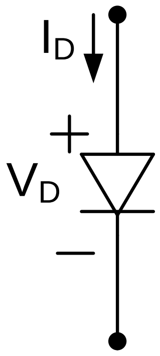
What variables available in a process have units volts?

V_{DD}, V_T, V_D (diode) ,V_Z,V_{BE},V_t ,V_{TH} ???

What variables which have units volts satisfy the desired properties of a voltage reference?

How can a circuit be designed that “expresses” the desired variables?

Voltage References



Consider the Diode

$$I_D = J_S A e^{\frac{V_D}{V_t}}$$

$$J_S = \tilde{J}_{SX}\left[T^m e^{\frac{-V_{G0}}{V_t}}\right]$$

$$V_t = \frac{kT}{q}$$

$$\frac{k}{q} = \frac{1.38 \mathrm{x} 10^{-23}}{1.602 \mathrm{x} 10^{-19}} \frac{\mathrm{V}}{{}^{o}\mathrm{K}} = 8.614 \mathrm{x} 10^{-5} \frac{\mathrm{V}}{{}^{o}\mathrm{K}}$$

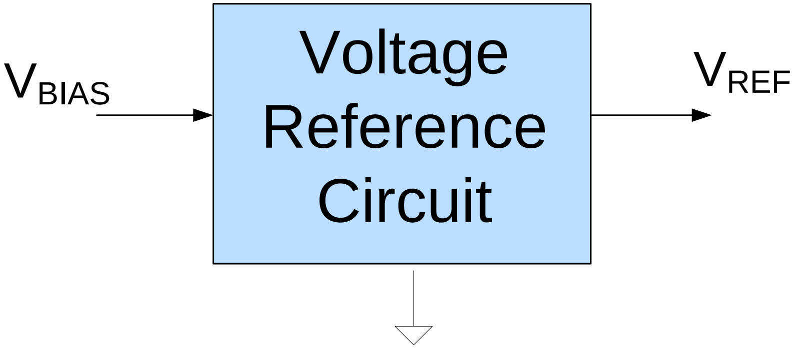
$$V_{G0} = 1.206\mathrm{V}$$

termed the bandgap voltage

pn junction characteristics highly temperature dependent through both the exponent and J_S

V_{G0} is nearly independent of process and temperature

Voltage References



Observation – Variables with units Volts needed to build any voltage reference

What variables available in a process have units volts?

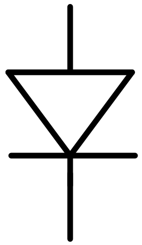
V_{DD}, V_T, V_D (diode) ,V_Z,V_{BE},V_t ,V_{G0} ???

What variables which have units volts satisfy the desired properties of a voltage reference? V_{G0} and ??

How can a circuit be designed that "expresses" the desired variables?

V_{G0} is deeply embedded in a device model with horrible temperature effects !
Good diodes are not widely available in most MOS processes !

Voltage References



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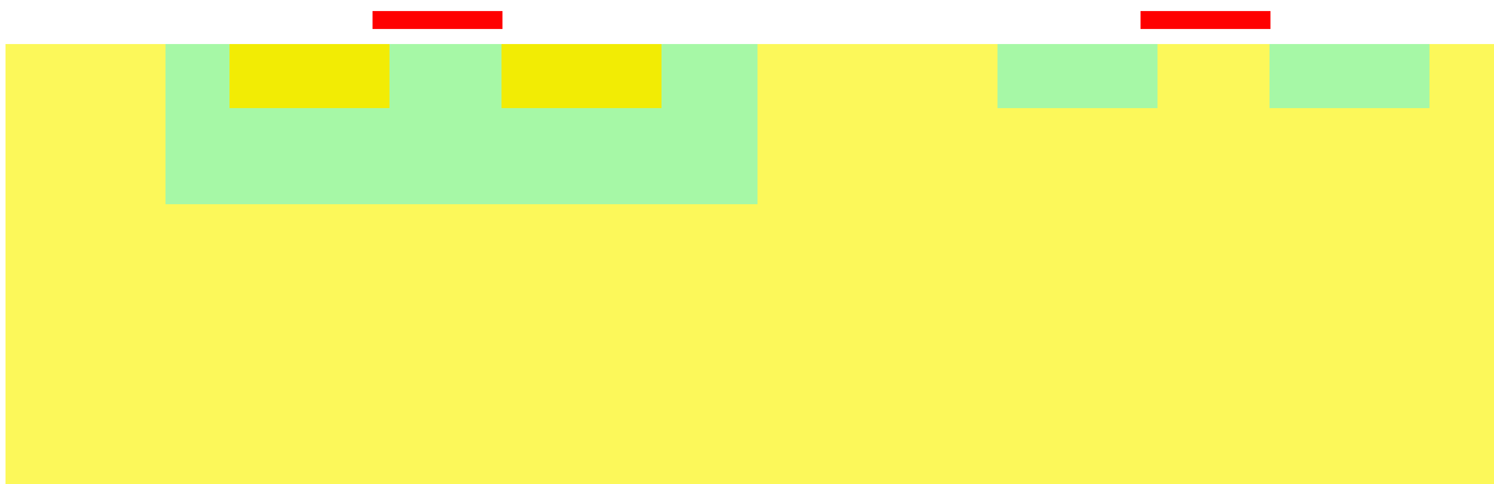
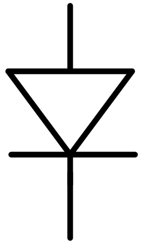


Voltage References



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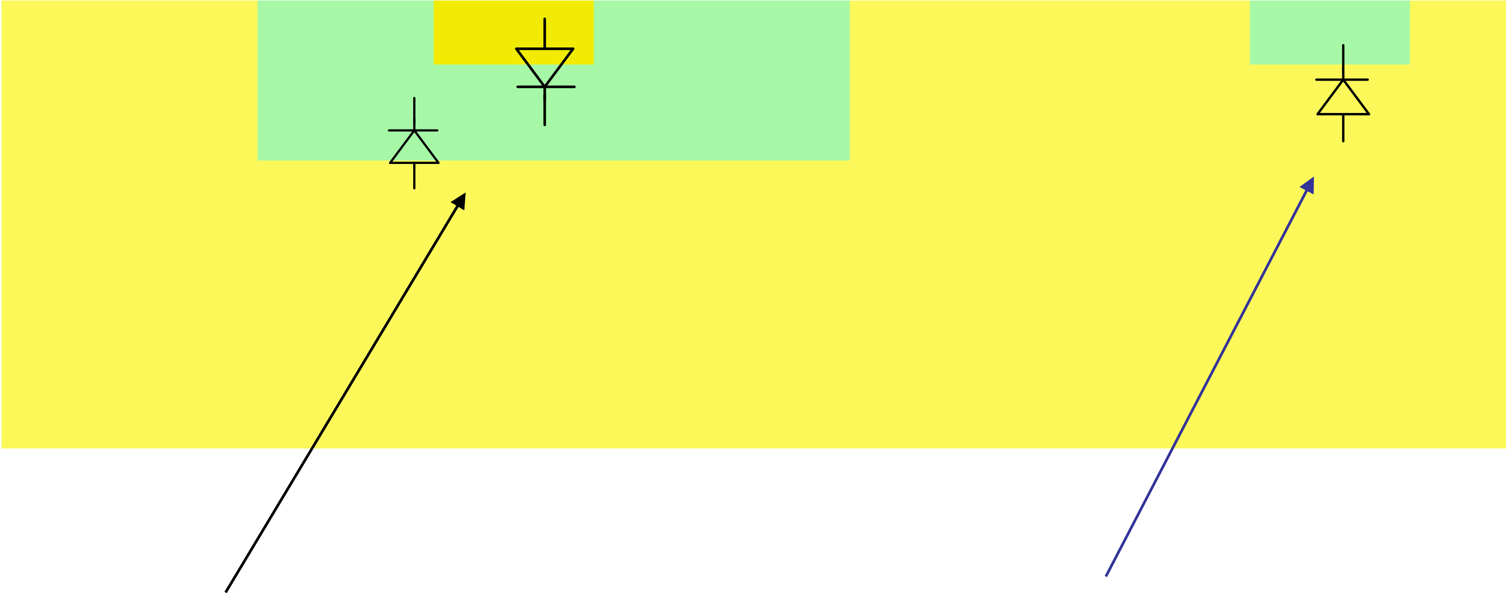
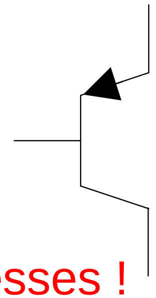


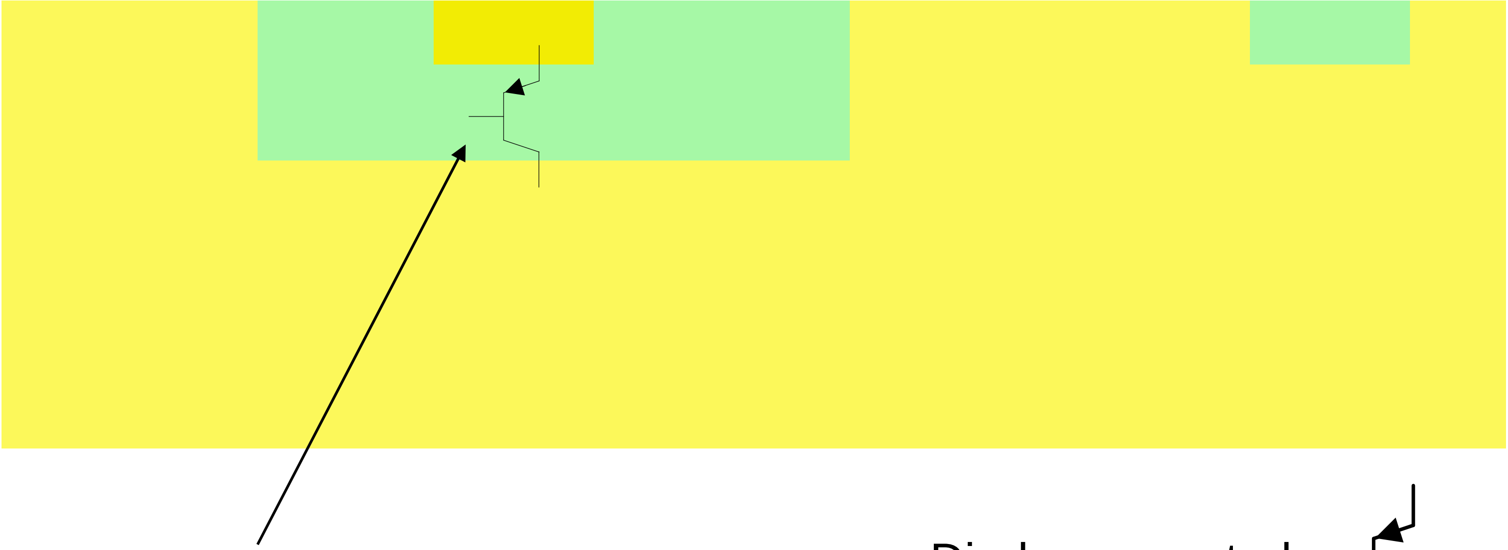
These diodes interact and
actually form substrate pnp transistor

Not practical to forward bias junction

Voltage References

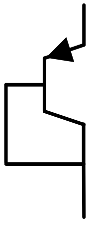


Good diodes are not widely available in most MOS processes !

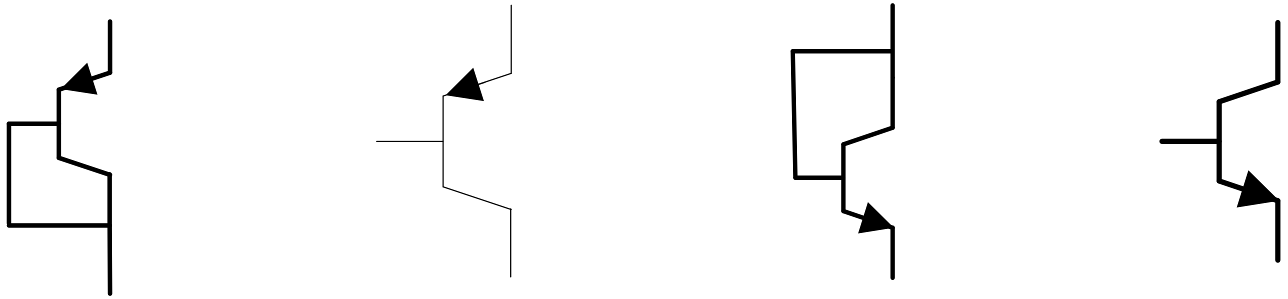


Substrate pnp transistor

Diode-connected
substrate pnp



Voltage References



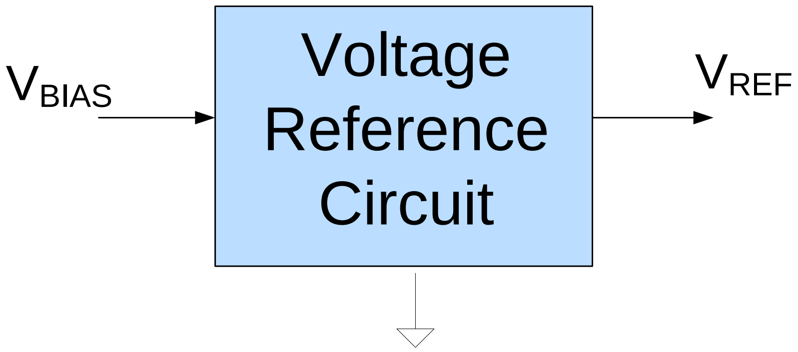
$$I_C = J_S A e^{\frac{V_{BE}}{V_t}}$$

$$J_S = \tilde{J}_{SX}\left[T^m e^{\frac{-V_{G0}}{V_t}}\right]$$

Bandgap Voltage Appears in BJT Model Equation as well

$$I_C(T) = \left(\tilde{J}_{SX} A\left[T^m e^{\frac{-V_{G0}}{V_t}}\right]\right) e^{\frac{V_{BE}(T)}{V_t}}$$

Voltage References

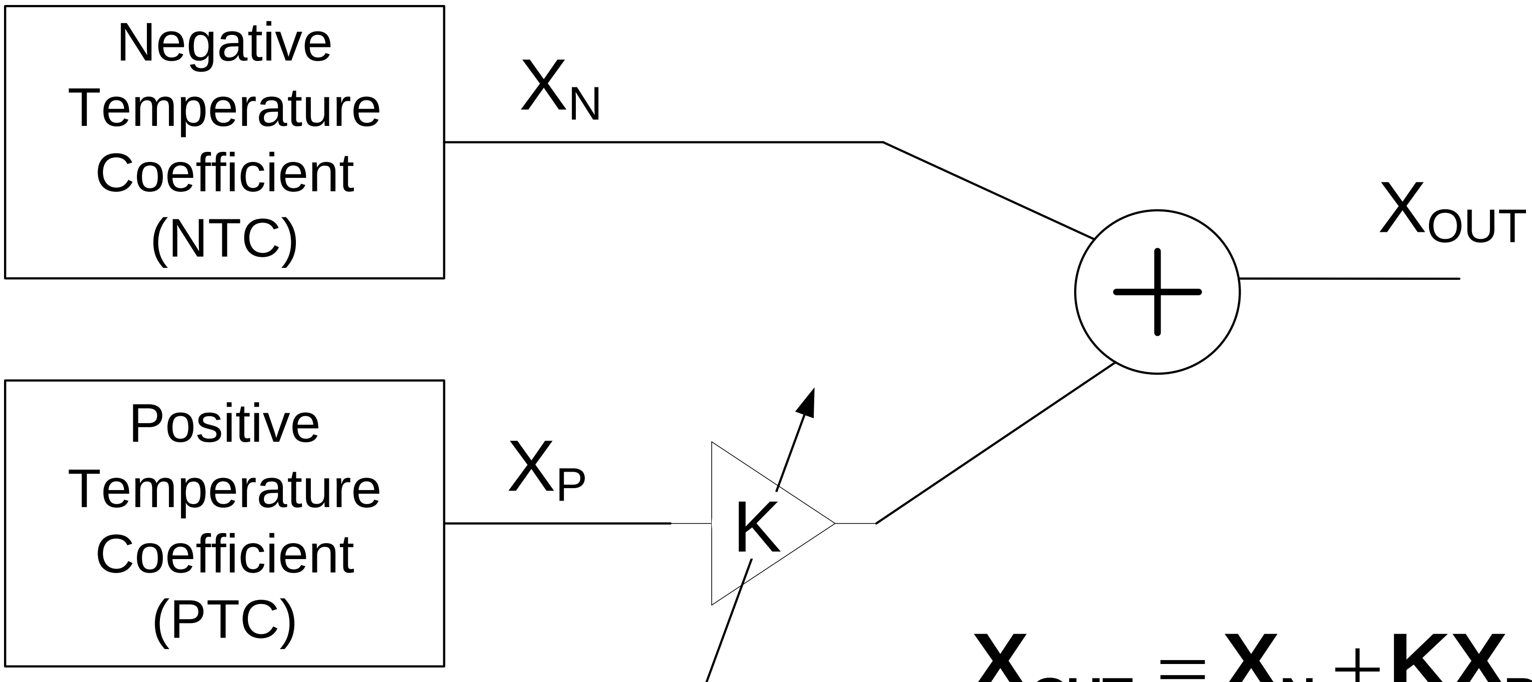


Voltage references that "express" the bandgap voltage are termed "Bandgap References"

V_{G0} is deeply embedded in a device model with horrible temperature effects !

Good BJTs are not widely available in most MOS processes but the substrate pnp is available !

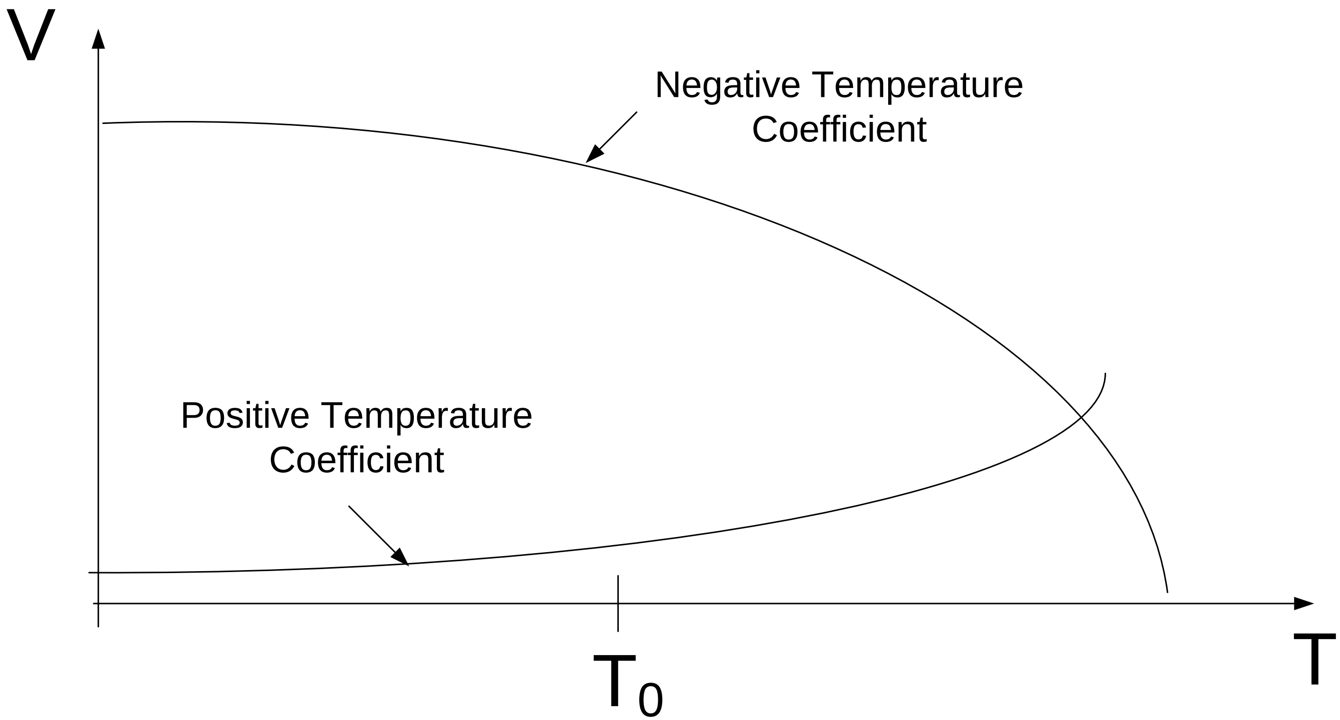
Standard Approach to Building Voltage References



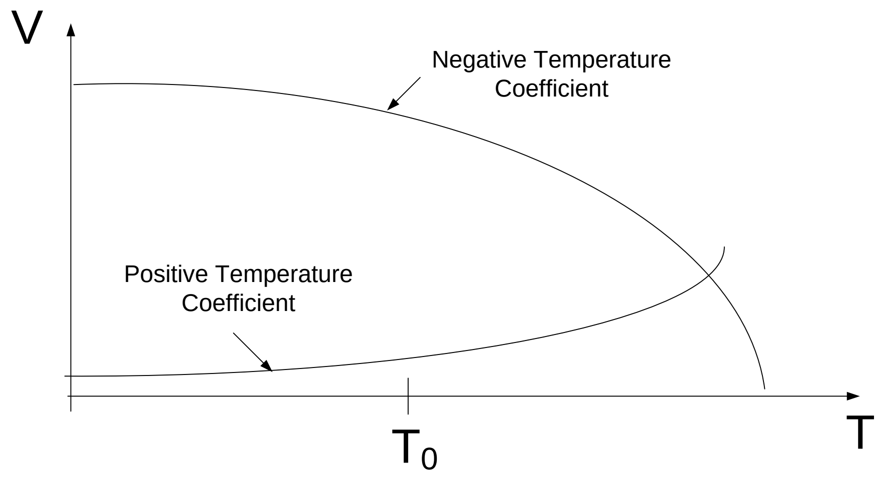
$$X_{OUT} = X_N + KX_P$$

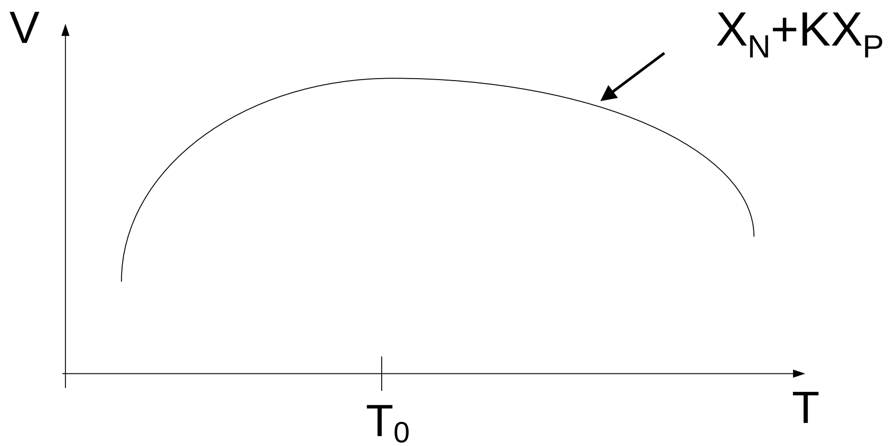
Pick K so that at some temperature T_0, $\left.\frac{\partial(X_N + KX_P)}{\partial T}\right|_{T=T_0} = 0$

Standard Approach to Building Voltage References



Standard Approach to Building Voltage References



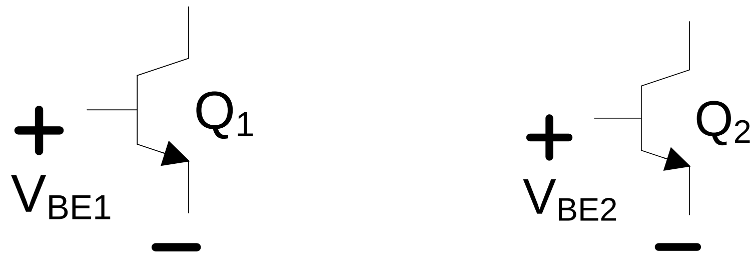


Select K so that

$$\left.\frac{\partial\left(X_N + KX_P\right)}{\partial T}\right|_{T=T_0} = 0$$

Bandgap Voltage References

Consider two BJTs (or diodes)



$$I_C(T) = \left(\tilde{I}_{SX} \left[T^m e^{\frac{-V_{G0}}{V_t}} \right] \right) e^{\frac{V_{BE}(T)}{V_t}}$$

$$V_{BE} = V_{G0} + V_t \ln\left(\frac{I_C}{\tilde{J}_{SX} A_E} \right) - mV_t \ln T$$

$$V_{BE2} - V_{BE1} = \Delta V_{BE} = \left[\frac{k}{q} \ln\left(\frac{I_{C2}}{I_{C1}} \frac{A_{E1}}{A_{E2}} \right) \right] T$$

If the $\frac{I_{C2} A_{E1}}{I_{C1} A_{E2}}$ ratio is constant and >1, the TC of ΔV_{BE} is positive

ΔV_{BE} is termed a PTAT voltage (Proportional to Absolute Temperature)

This relationship applies irrespective of how temperature dependent I_{C1} and I_{C2} may be provided the ratio is constant !!

Bandgap Voltage References

Consider two BJTs (or diodes)

$$V_{BE2} - V_{BE1} = \Delta V_{BE} = \left[\frac{k}{q}\ln\left(\frac{I_{C2}A_{E1}}{I_{C1}A_{E2}}\right)\right]T$$

$$\frac{\partial\left(V_{BE2} - V_{BE1}\right)}{\partial T} = \frac{k}{q}\ln\left(\frac{I_{C2}A_{E1}}{I_{C1}A_{E2}}\right)$$

At room temperature if $\ln\left(\frac{I_{C2}A_{E1}}{I_{C1}A_{E2}}\right) = 1$

$$V_{BE2} - V_{BE1} = \left[8.6x10^{-5}x300\right] = 25.8mV$$

and

$$\left.\frac{\partial\left(V_{BE2} - V_{BE1}\right)}{\partial T}\right|_{T=T_0=300^o K} = 8.6x10^{-5} = 86\mu V/^o C$$

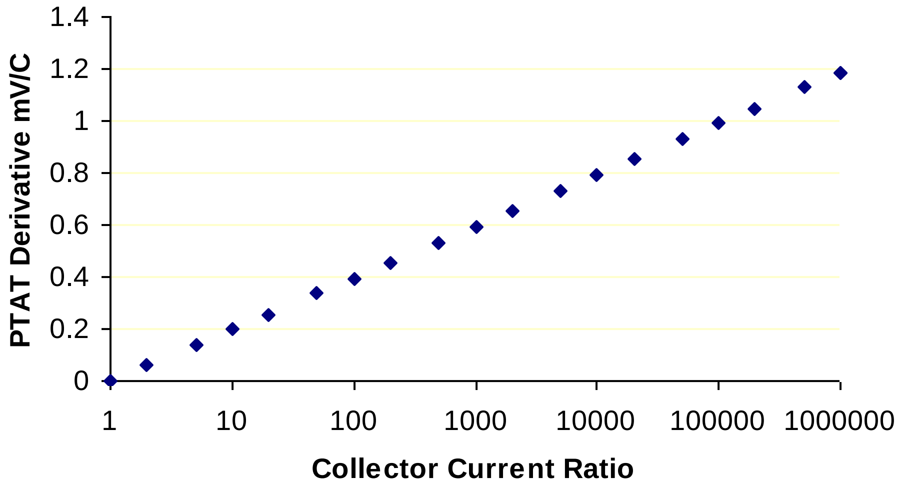
The temperature coefficient of the PTAT voltage is rather small

Bandgap Voltage References

Consider two BJTs (or diodes)

$$\frac{\partial\left(V_{BE2}-V_{BE1}\right)}{\partial T}=\frac{k}{q}\ln\left(\frac{I_{C2}}{I_{C1}}\right)$$

At room temperature if $A_{E1}=A_{E2}$



The temperature coefficient of the PTAT voltage is rather small even if large collector current ratios are used

Bandgap Voltage References

Consider two BJTs (or diodes) Assume m=2.3, V_{G0}=1.2V

$$I_C(T) = \left(\tilde{J}_{SX} A_E \left[T^m e^{\frac{-V_{G0}}{V_t}} \right] \right) e^{\frac{V_{BE}(T)}{V_t}}$$

$$V_{BE} = V_{G0} + V_t \ln\left(\frac{I_C}{\tilde{J}_{SX} A_E} \right) - mV_t \ln T$$

If I_C is independent of temperature, it follows that

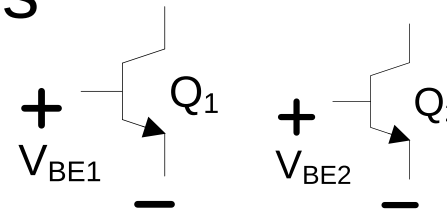
$$\frac{\partial V_{BE}}{\partial T} = \frac{k}{q}\left[-m + \left(\frac{V_{BE} - V_{G0}}{V_t} \right) \right]$$

$$\left. \frac{\partial V_{BE}}{\partial T} \right|_{T=T_0=300^o K} \cong 8.6x10^{-5}\left[-2.3 + \left(\frac{0.65 - 1.2}{25mV} \right) \right] \cong -2.1mV/^o C$$

Bandgap Voltage References

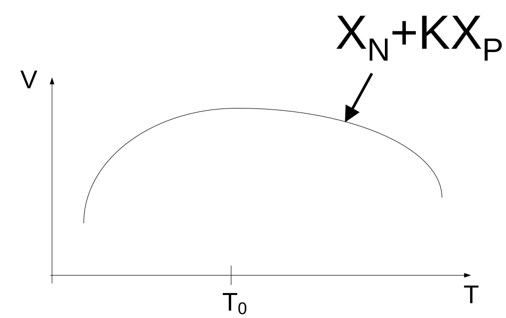
Consider two BJTs (or diodes)

Assume m=2.3, V_{G0}=1.2V



Thus if I_C independent of temperature

$$\left.\frac{\partial V_{BE}}{\partial T}\right|_{T=T_0=300^oK} \cong -2.1\text{mV}/{}^oC$$



And if $\ln\left(\frac{I_{C2}A_{E1}}{I_{C1}A_{E2}}\right)=1$

$$\left.\frac{\partial\left(V_{BE2}-V_{BE1}\right)}{\partial T}\right|_{T=T_0=300^oK} = 86\mu V/{}^oC$$

Magnitude of TC of PTAT source is much smaller than that of V_{BE} source

Define:

$X_N=V_{BE}$ $X_P=V_{BE2}-V_{BE1}$

Create circuit with:

$$\mathbf{X_{OUT}=X_N+KX_P}$$

If we want $\left.\frac{\partial\left(\mathbf{X_N}+\mathbf{KX_P}\right)}{\partial \mathbf{T}}\right|_{\mathbf{T=T_0}} = \mathbf{0}$ K will need to be large

Bandgap Voltage References

Consider two BJTs (or diodes)

It was just shown that if I_C is independent of temperature

$$\left.\frac{\partial V_{BE}}{\partial T}\right|_{T=T_0=300^oK} \cong 8.6x10^{-5}\left[-2.3+\left(\frac{0.65-1.2}{25mV}\right)\right] \cong -2.1mV/^oC$$

If I_C is reasonably independent of temperature, V_{BE} will still provide a negative TC

$$V_{BE}=V_{G0}+V_t\ln\left(\frac{I_C}{\tilde{J}_{SX}A_E}\right)-mV_t\ln T$$

Even if I_C is highly dependent on temperature, V_{BE} will still provide a negative TC

Bandgap Voltage References

Consider two BJTs (or diodes)

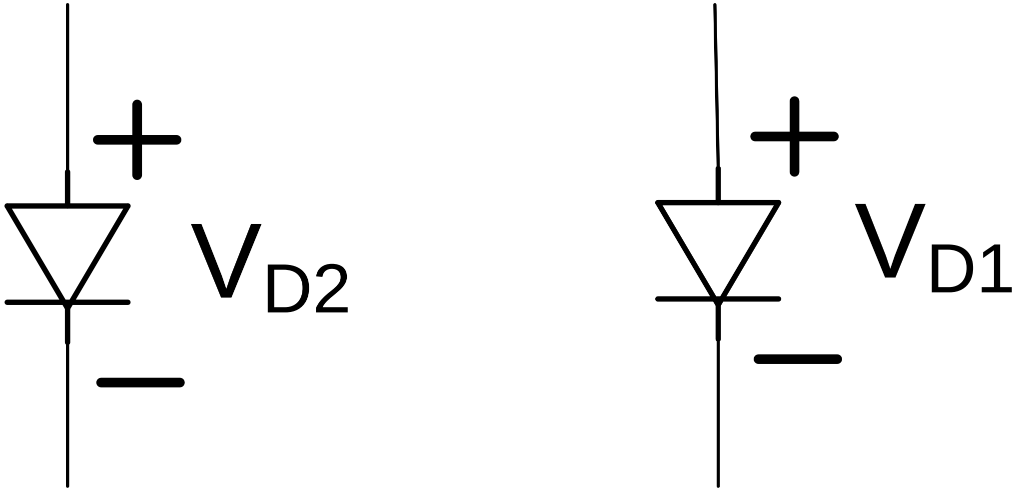
Key observation about diodes and diode-connected BJTs

1. If ratio of currents in two devices is constant, ΔV_{BE} is PTAT independent of the temperature dependence of the currents and sensitivity is small

2. VBE has a negative temperature coefficient for a wide range of temperature dependent or temperature independent currents and sensitivity is much larger than that of ΔV_{BE}

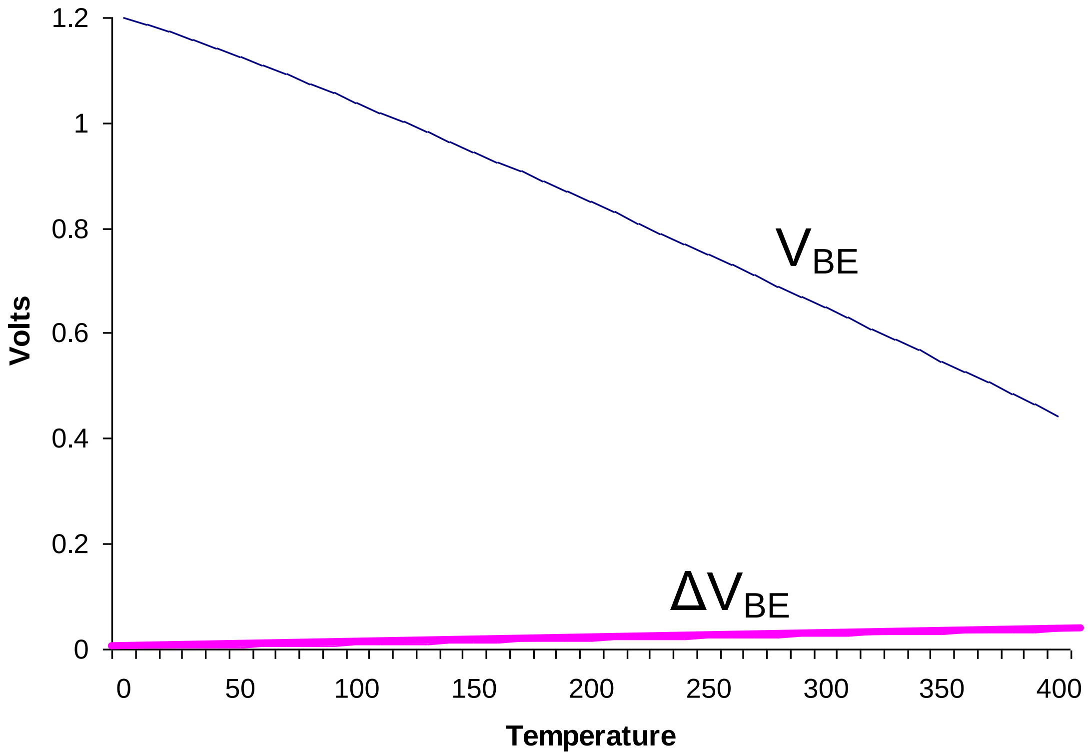
Bandgap Reference Circuits

- Circuits that implement ΔV_{BE} and V_{BE} or ΔV_D and V_D widely used to build bandgap references



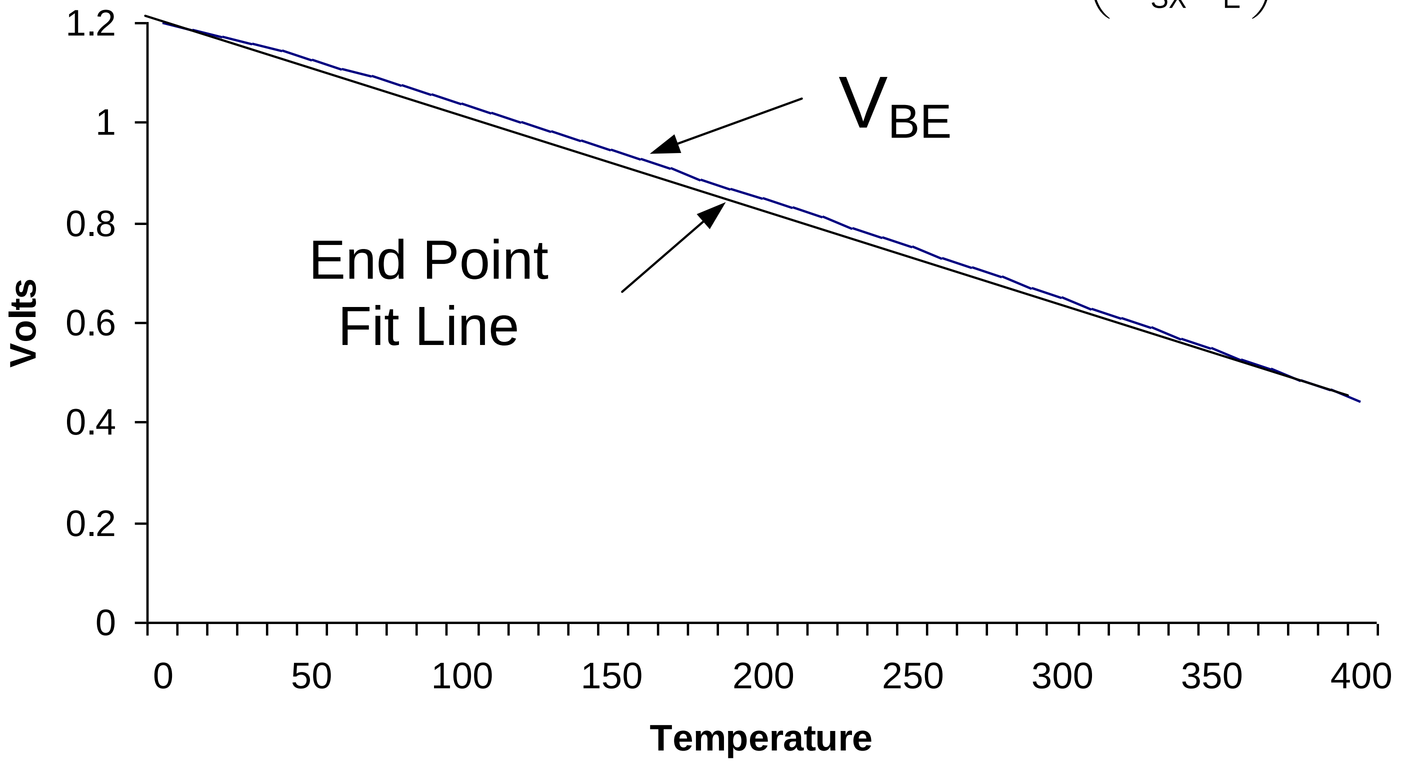
V_{BE} and ΔV_{BE} with constant I_C

$$V_{BE} = V_{G0} + V_t \ln\left(\frac{I_C}{\tilde{J}_{SX} A_E}\right) - mV_t \ln T$$



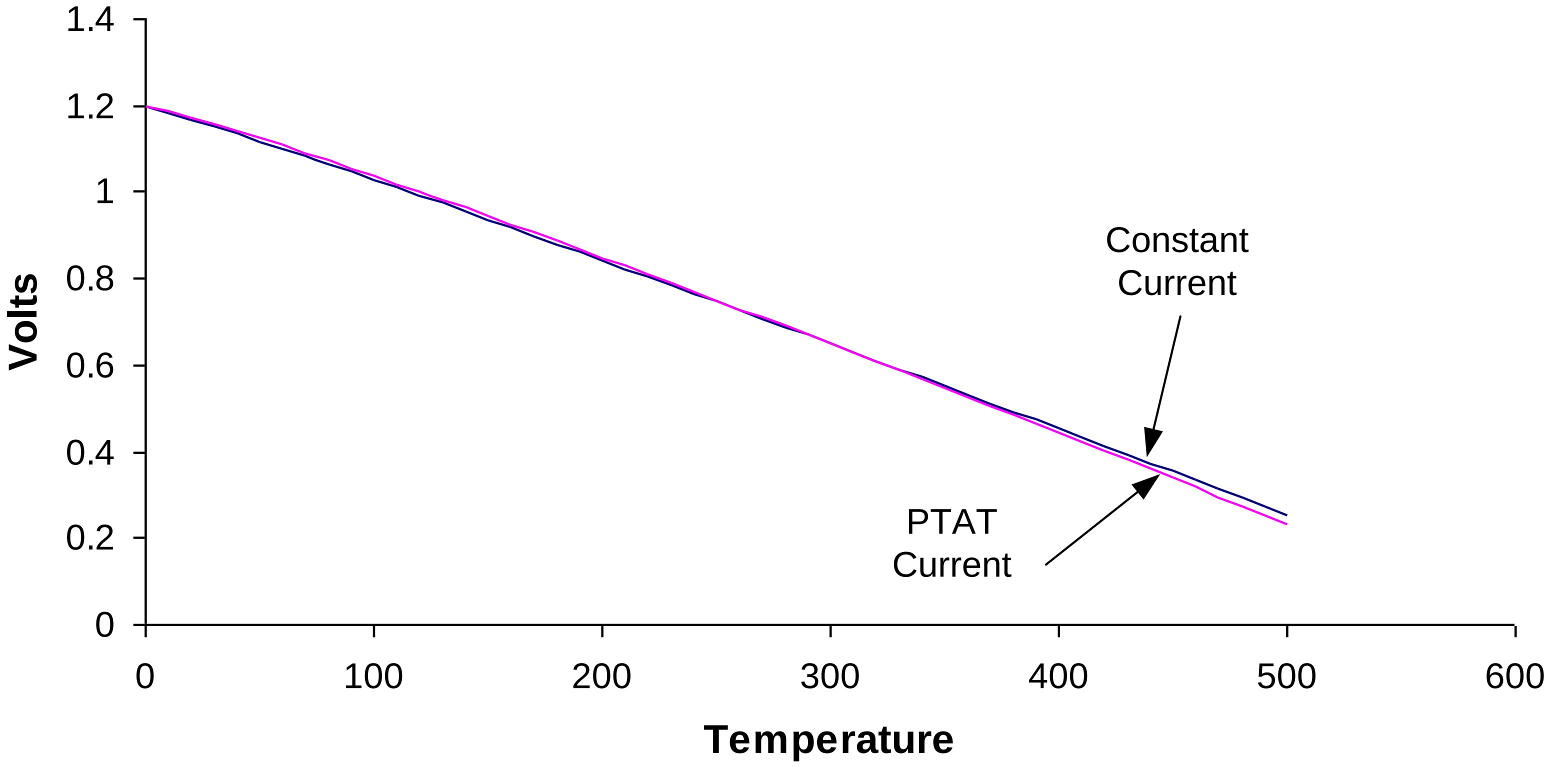
V_{BE} plot for constant I_C

$$V_{BE} = V_{G0} + V_t \ln\left(\frac{I_C}{\tilde{J}_{SX} A_E}\right) - mV_t \ln T$$



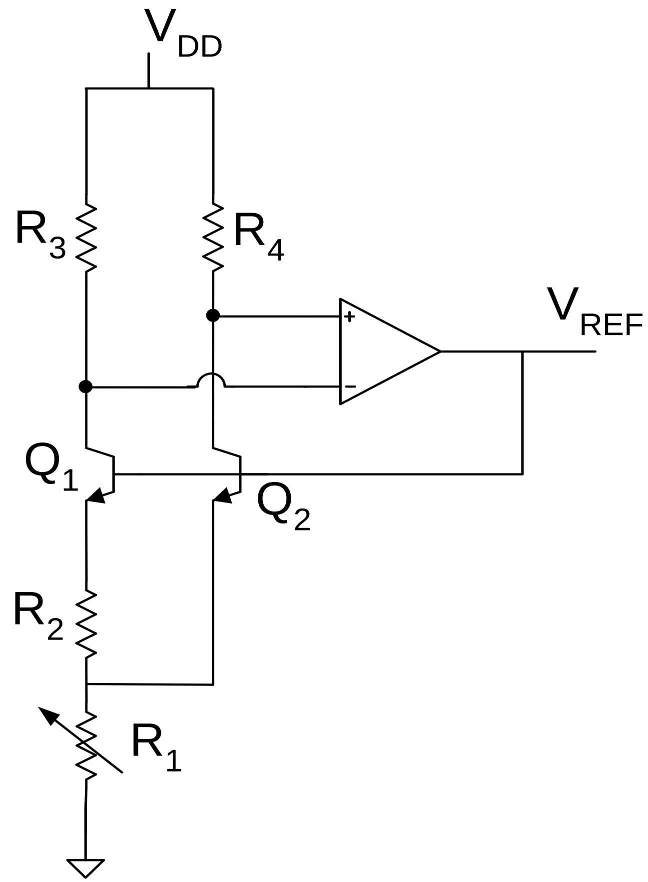
Combined effects of the T and TlnT terms in V_{BE} is nearly linear dependent on T

Comparison of V_{BE} with constant current and PTAT current



Even if I_C is highly-dependent on current, temperature dependence of V_{BE} is still nearly linearly dependent upon T

First Bandgap Reference (and still widely used!)



P.Brokaw, "A Simple Three-Terminal IC Bandgap Reference", IEEE Journal of Solid State Circuits, Vol. 9, pp. 388-393, Dec. 1974.

- Brokaw coined term "bandgap reference" when referring to this circuit
- Properties very similar to a circuit introduced by Widlar a small while earlier

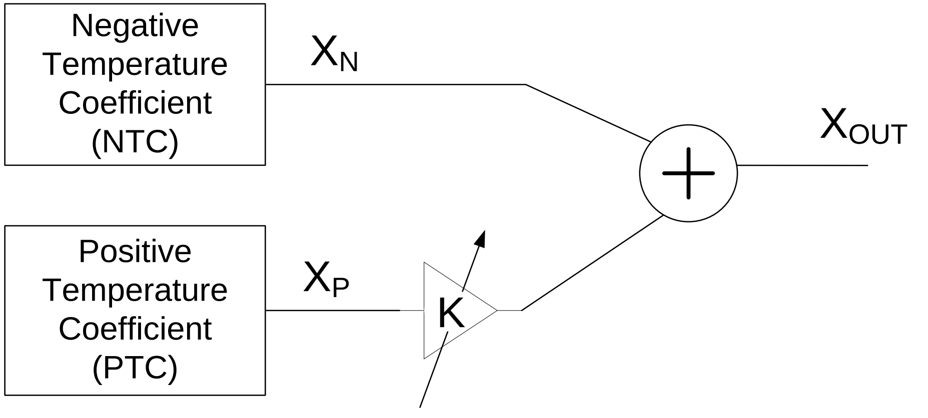
Most Published Analysis of Bandgap Circuits

V_{REF} often expressed as:

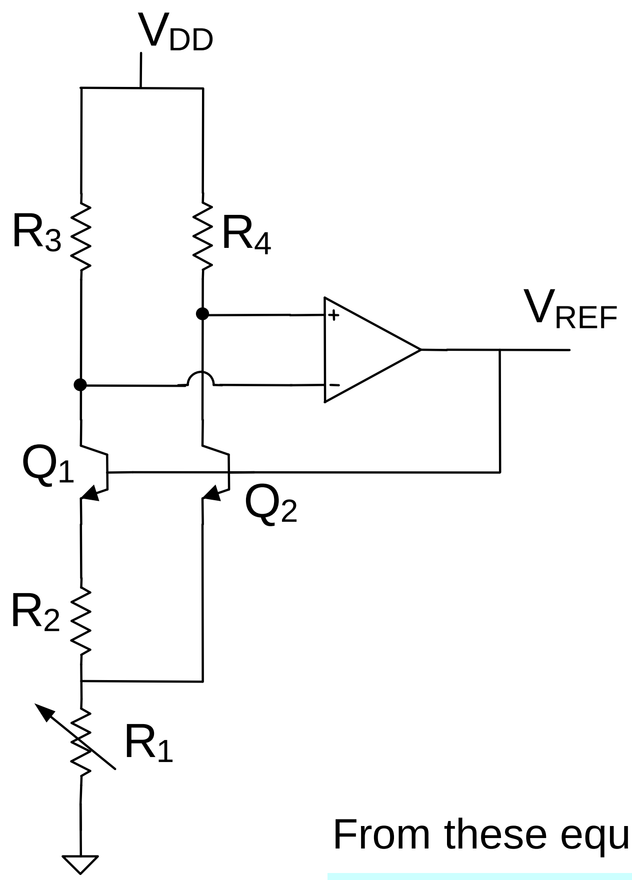
$$V_{REF}=V_{G0}+\frac{T}{T_0}\left(V_{BE0}-V_{G0}\right)+K\frac{kT}{q}\ln\left(\frac{J_2}{J_1}\right)+(m-1)\frac{kT}{q}\ln\left(\frac{T_0}{T}\right)$$

where K is the gain of the PTAT signal

(Not a solution and dependent upon both T_0 and V_{BE0})



First Bandgap Reference (and still widely used!)



$$I_{E1}R_2 + V_{BE1} = V_{BE2}$$

$$V_{REF} = V_{BE2} + (I_{E1} + I_{E2})R_1$$

$$I_{C1} = \frac{V_{DD} - V_{C2}}{R_3}$$

$$I_{C2} = \frac{V_{DD} - V_{C2}}{R_4}$$

$$I_{C1} = \alpha_1 I_{E1}$$

$$I_{C2} = \alpha_2 I_{E2}$$

$$I_{E1} = I_{E2}\left[\frac{\alpha_2}{\alpha_1}\frac{R_4}{R_3}\right] \rightarrow I_{C1} = I_{C2}\left[\frac{R_4}{R_3}\right]$$

$$\alpha = \frac{\beta}{1+\beta}$$

From these equations can show

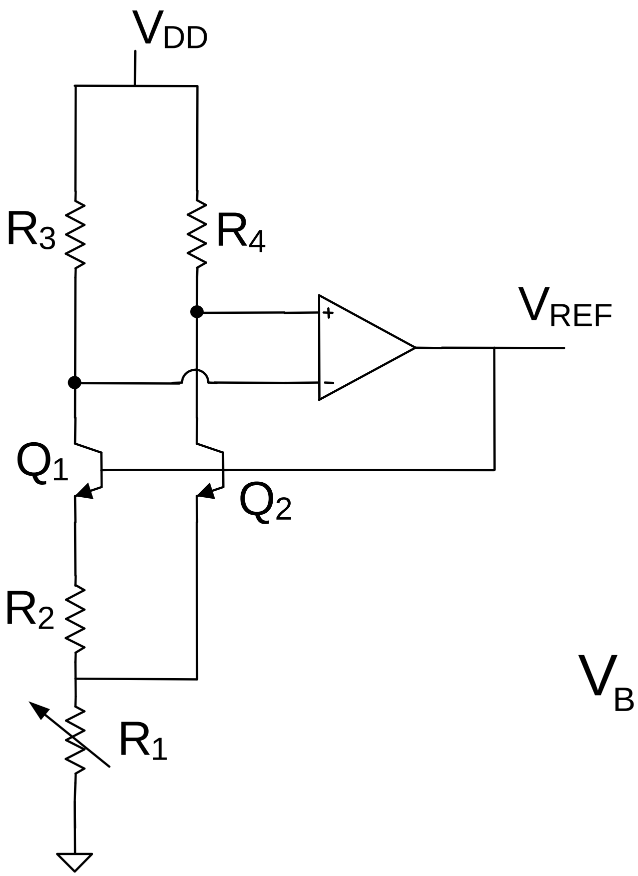
$$V_{REF} = V_{BE2} + (V_{BE2} - V_{BE1})\left[\frac{R_1}{R_2}\left(1 + \frac{\alpha_1}{\alpha_2}\frac{R_3}{R_4}\right)\right]$$

Not a solution but can provide zero temp slope by adjusting R_1

First Bandgap Reference (and still widely used!)

Will now obtain solution for V_{REF} (in terms of component values and model parameters)

$$V_{REF} = V_{BE2} + \left(V_{BE2} - V_{BE1}\right)\left[\frac{R_1}{R_2}\left(1+\frac{\alpha_1}{\alpha_2}\frac{R_3}{R_4}\right)\right]$$



$$V_{BE1}=V_{G0}+V_t\ln\left(\frac{I_{C1}}{\tilde{J}_{SX}A_{E1}}\right)-mV_t\ln T$$

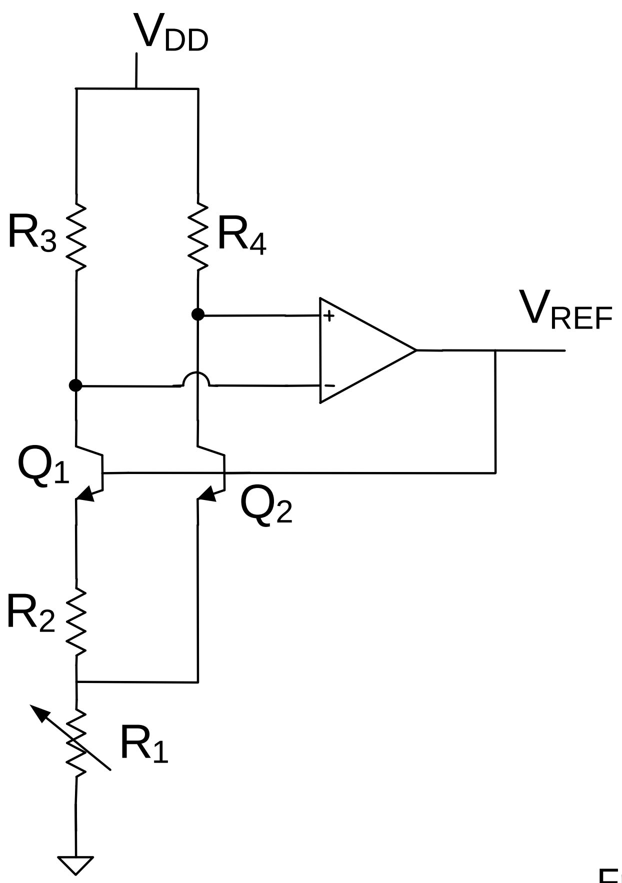
$$V_{BE2}=V_{G0}+V_t\ln\left(\frac{I_{C2}}{\tilde{J}_{SX}A_{E2}}\right)-mV_t\ln T$$

$$I_{C1} = I_{C2}\left[\frac{R_4}{R_3}\right]$$

$$V_{BE2}\text{-}V_{BE1}=\Delta V_{BE}=\left[\frac{k}{q}\ln\left(\frac{A_{E1}}{A_{E2}}\left[\frac{R_3}{R_4}\right]\right)\right]T$$

First Bandgap Reference (and still widely used!)

Will now obtain solution for V_{REF} (in terms of component values and model parameters)



$$V_{REF} = V_{BE2} + (V_{BE2} - V_{BE1})\left[\frac{R_1}{R_2}\left(1+\frac{\alpha_1}{\alpha_2}\frac{R_3}{R_4}\right)\right]$$

$$V_{BE1} = V_{G0} + V_t \ln\left(\frac{I_{C1}}{\tilde{J}_{SX} A_{E1}}\right) - mV_t \ln T$$

$$V_{BE2} = V_{G0} + V_t \ln\left(\frac{I_{C2}}{\tilde{J}_{SX} A_{E2}}\right) - mV_t \ln T$$

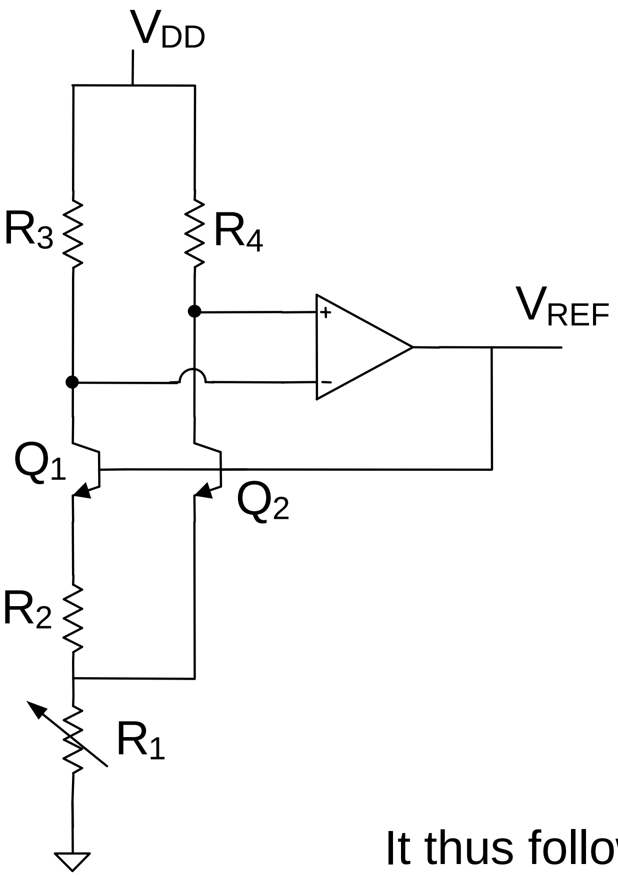
$$I_{C1} = I_{C2}\left[\frac{R_4}{R_3}\right]$$

$$\frac{I_{C1}}{\alpha_1} R_2 + V_{BE1} = V_{BE2}$$

From the expression for V_{BE2} and some routine but tedious manipulations it follows that

$$V_{BE2} = V_{G0} + (1-m)V_t \ln T + V_t \ln\left(\frac{k}{q}\frac{\alpha_1}{R_2 A_{E2} \tilde{J}_{SX}}\frac{R_3}{R_4}\ln\left(\frac{A_{E1}}{A_{E2}}\frac{R_3}{R_4}\right)\right)$$

First Bandgap Reference (and still widely used!)



$$V_{REF} = V_{BE2} + \left(V_{BE2} - V_{BE1}\right)\left[\frac{R_1}{R_2}\left(1+\frac{\alpha_1}{\alpha_2}\frac{R_3}{R_4}\right)\right]$$

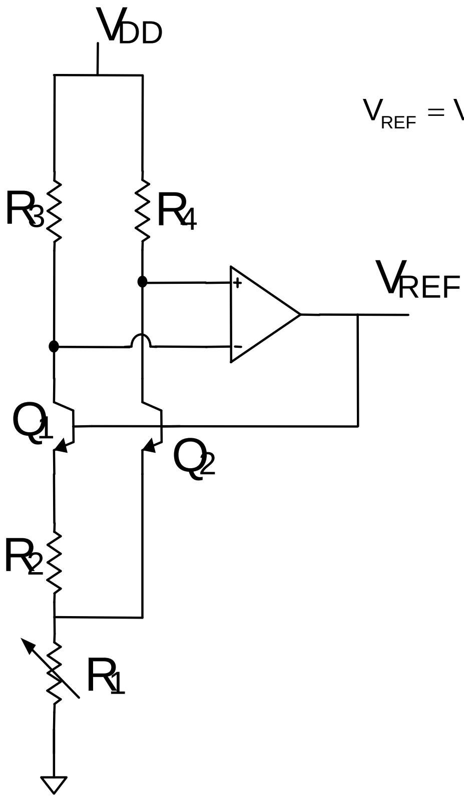
$$V_{BE2}\text{-}V_{BE1} = \left[\frac{k}{q}\ln\left(\frac{A_{E1}}{A_{E2}}\left[\frac{R_3}{R_4}\right]\right)\right]T$$

$$V_{BE2} = V_{G0} + \left(1-m\right)V_t \ln T \; + V_t \ln\left(\frac{k}{q}\frac{\alpha_1}{R_2 A_{E2}\tilde{J}_{SX}}\frac{R_3}{R_4}\ln\left(\frac{A_{E1}}{A_{E2}}\frac{R_3}{R_4}\right)\right)$$

It thus follows that:

$$V_{REF} = V_{G0} + V_t \ln\left\{\frac{\alpha_1}{R_2}\frac{R_3}{R_4}T\frac{k}{q}\ln\left(\frac{A_{E1}}{A_{E2}}\frac{R_3}{R_4}\right)\right\} - V_t\left(\ln\left(\tilde{I}_{SX2}\right) + m\ln T\right) + \left[\frac{k}{q}\ln\left(\frac{A_{E1}}{A_{E2}}\left(\frac{R_3}{R_4}\right)\right)\right]\left[\frac{R_1}{R_2}\left(1+\frac{\alpha_1}{\alpha_2}\frac{R_3}{R_4}\right)\right]T$$

First Bandgap Reference (and still widely used!)



$$V_{REF} = V_{BE2} + \left(V_{BE2} - V_{BE1}\right)\left[\frac{R_1}{R_2}\left(1+\frac{\alpha_1}{\alpha_2}\frac{R_3}{R_4}\right)\right]$$

$$V_{REF} = V_{G0} + V_t \ln\left\{\frac{\alpha_1}{R_2}\frac{R_3}{R_4} T \frac{k}{q} \ln\left(\frac{A_{E1}}{A_{E2}}\frac{R_3}{R_4}\right)\right\} - V_t\left(\ln\left(\tilde{I}_{SX2}\right) + m\ln T\right) + \left[\frac{k}{q}\ln\left(\frac{A_{E1}}{A_{E2}}\left(\frac{R_3}{R_4}\right)\right)\right]\left[\frac{R_1}{R_2}\left(1+\frac{\alpha_1}{\alpha_2}\frac{R_3}{R_4}\right)\right] T$$

This can be expressed after some manipulations as

$$V_{REF} = a_1 + b_1 T + c_1 T \ln T$$

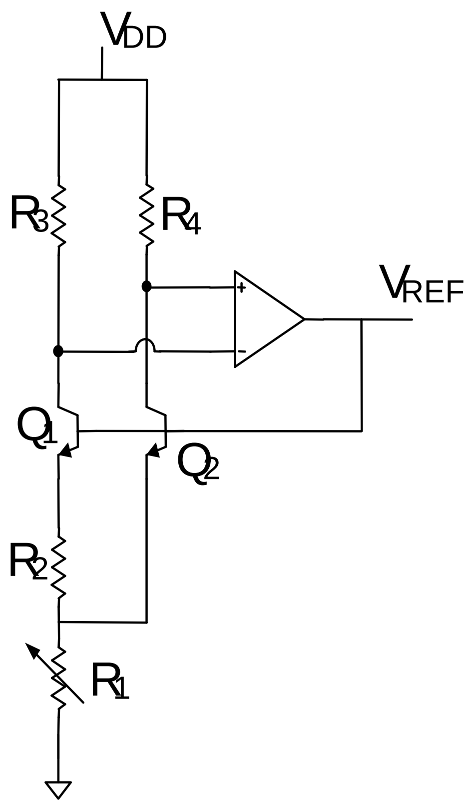
where

$\mathbf{a_1} = \mathbf{V_{GO}}$

$$b_1 = \frac{k}{q}\left(\frac{R_1}{R_2}\left(1+\frac{R_3\alpha_1}{R_4\alpha_2}\right)\ln\left(\frac{R_3}{R_4}\frac{A_{E1}}{A_{E2}}\right) + \ln\left(\frac{k}{q}\frac{R_3}{R_4}\alpha_1 \frac{\ln\left(\frac{R_3}{R_1}\frac{A_{E1}}{A_{E2}}\right)}{\tilde{I}_{SK2} R_2}\right)\right)$$

$$c_1 = \frac{k}{q}(1-m)$$

First Bandgap Reference (and still widely used!)



$$V_{REF} = a_1 + b_1 T + c_1 T \ln T$$

$$\mathbf{a_1 = V_{GO}}$$

$$b_1 = \frac{k}{q}\left(\frac{R_1}{R_2}\left(1+\frac{R_3\alpha_1}{R_4\alpha_2}\right)\ln\left(\frac{R_3}{R_4}\frac{A_{E1}}{A_{E2}}\right)+\ln\left(\frac{k}{q}\frac{R_3}{R_4}\alpha_1\frac{\ln\left(\frac{R_3}{R_1}\frac{A_{E1}}{A_{E2}}\right)}{\tilde{I}_{SK2}R_2}\right)\right)$$

$$c_1 = \frac{k}{q}(1-m)$$

$$\frac{dV_{REF}}{dT} = b_1 + c_1(1+\ln T) = 0$$

$$T_{INF} = e^{-\left(1+\frac{b_1}{c_1}\right)}$$

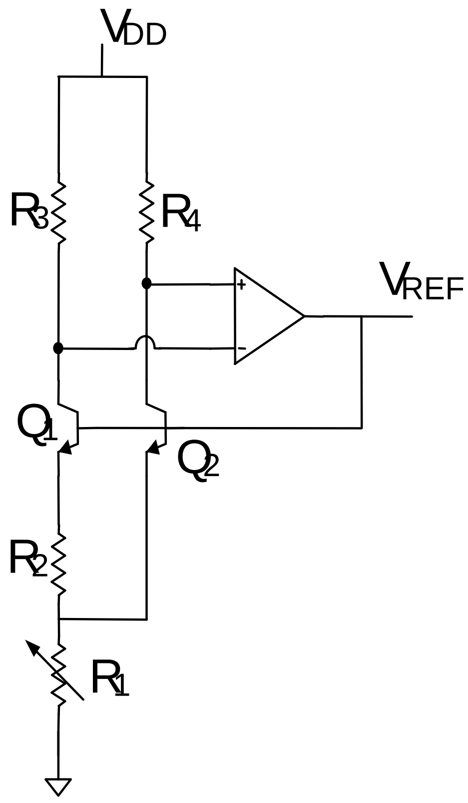
$$b_1 = -c_1(1+\ln T_{INF})$$

$$V_{REF} = a_1 - c_1 T_{INF}$$

$$V_{REF} = V_{G0} + \frac{kT_{INF}}{q}(m-1)$$

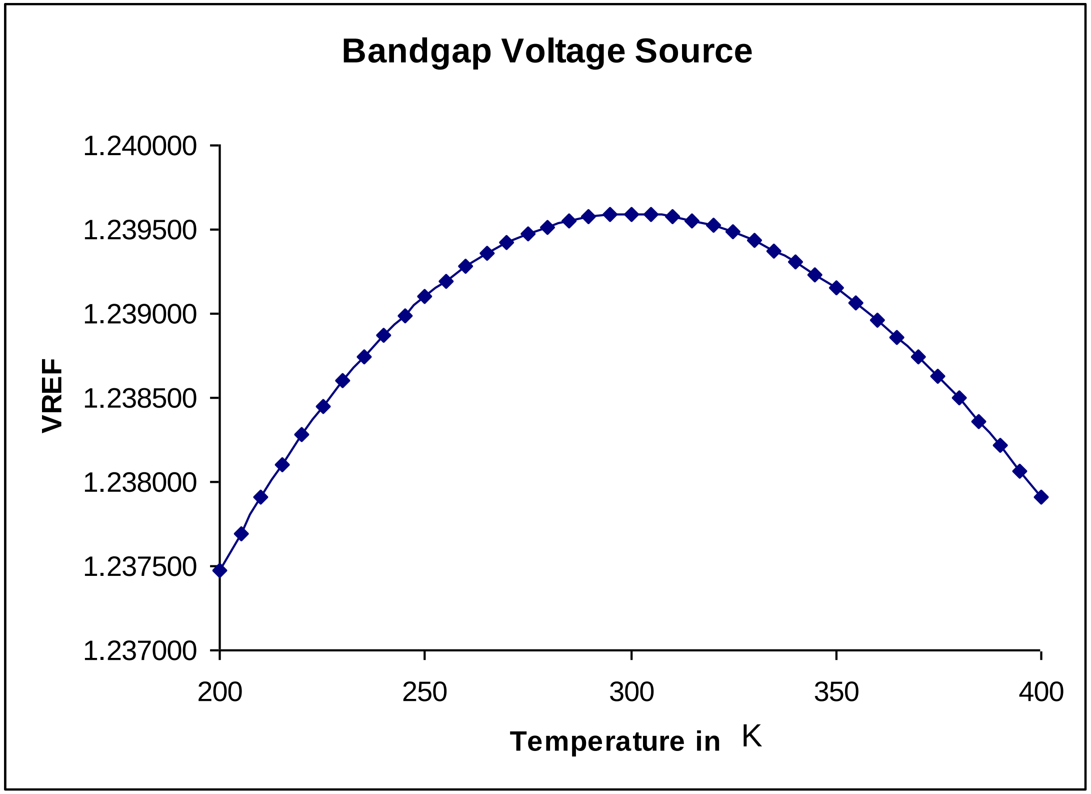
$\frac{kT_{INF}}{q}(m-1)$ is small ⟹ Nearly V_{GO} output at T_{INF}

First Bandgap Reference (and still widely used!)



$$V_{REF} = a_1 + b_1 T + c_1 T \ln T$$

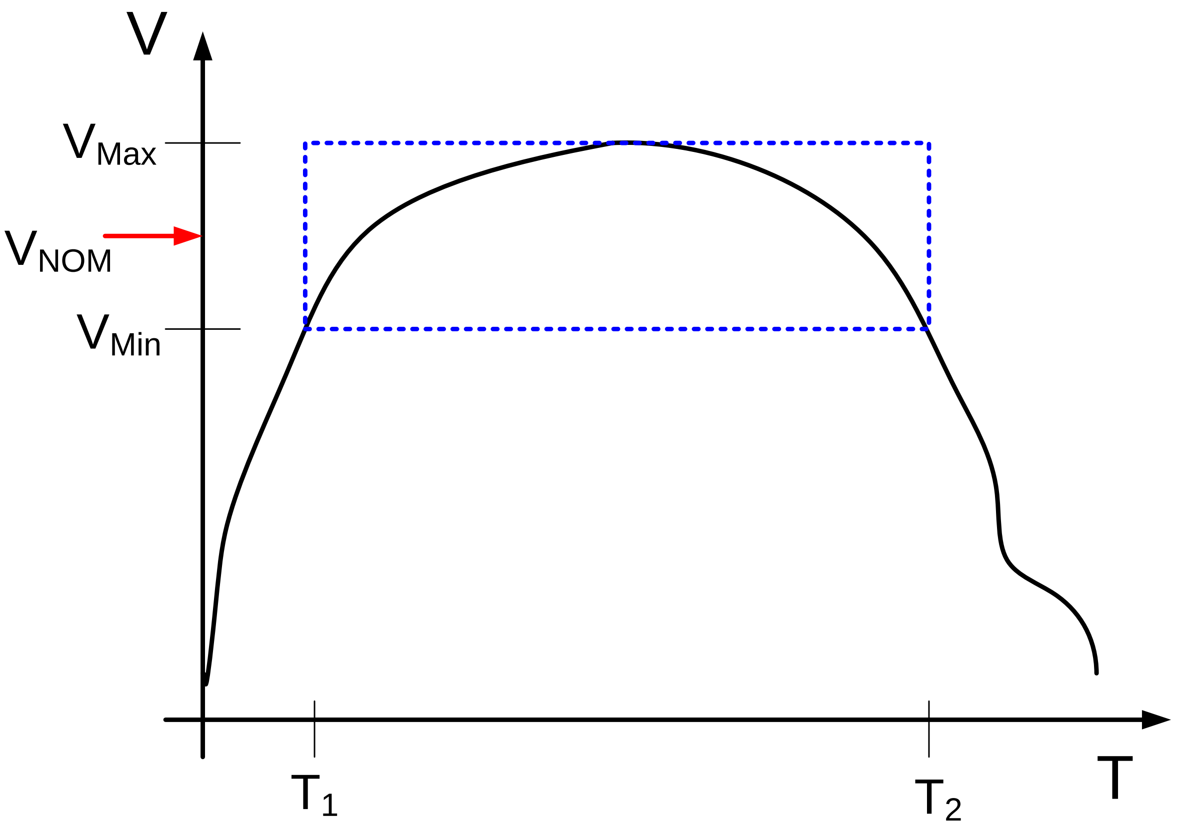
$$V_{REF}(T_{INF}) = V_{G0} + \frac{kT_{INF}}{q}(m-1)$$



VGO	1.206
TO	300
VBEO2	0.65
m-1	1.3
k/q	8.61E-05

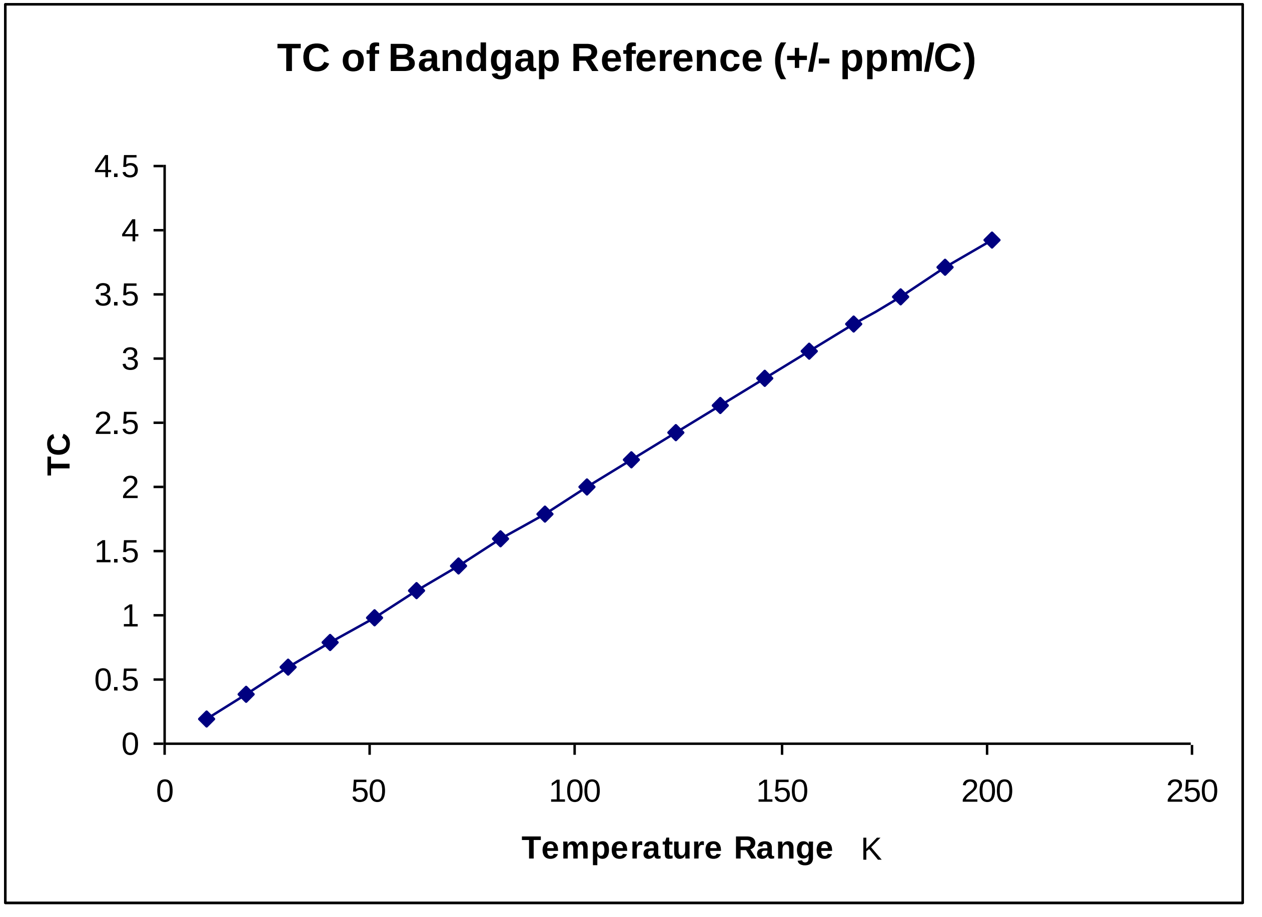
Only 2mV change over 200^{o}C temp range !

Temperature Coefficient



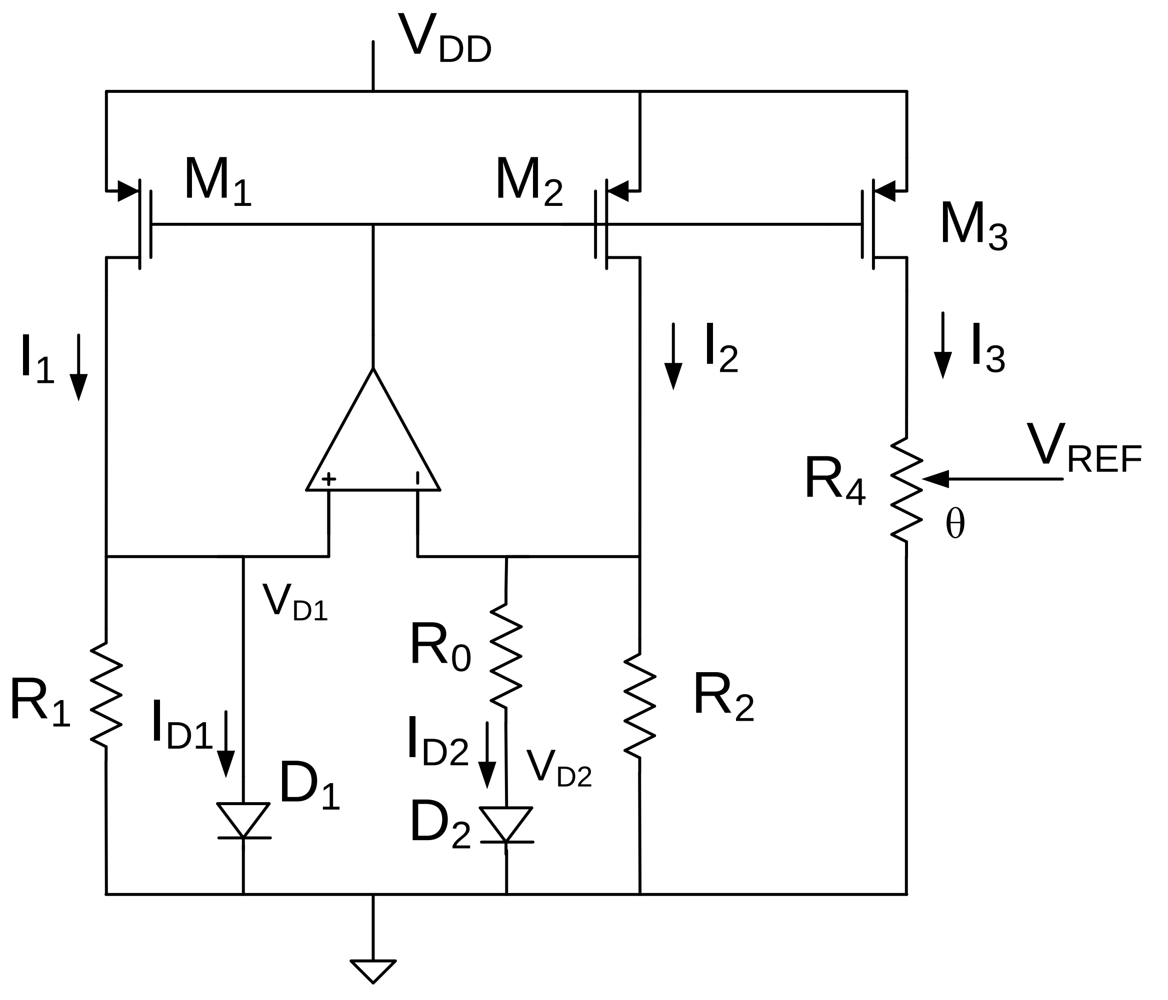
$$TC = \frac{V_{MAX} - V_{MIN}}{T_2 - T_1}$$

$$TC_{ppm} = \frac{V_{MAX} - V_{MIN}}{V_{NOM}(T_2 - T_1)} 10^6$$



TC of Bandgap Reference (+/- ppm/C)
TC
4.5
4
3.5
3
2.5
2
1.5
1
0.5
0
0
50
100
150
200
250
Temperature Range K

Bamba Bandgap Reference



$$\frac{I_{D2}}{I_{D1}} = \text{constant}$$

[7] H. Banba, H. Shiga, A. Umezawa, T. Miyaba, T. Tanzawa, A. Atsumi, and K. Sakkui, IEEE Journal of Solid-State Circuits, Vol. 34, pp. 670-674, May 1999.

Bamba Bandgap Reference

$$I_{R0} = \frac{\Delta V_{BE}}{R_0}$$

$$I_{R1} = \frac{V_{BE1}}{R_1}$$

$$I_{R2} = I_{R1}$$

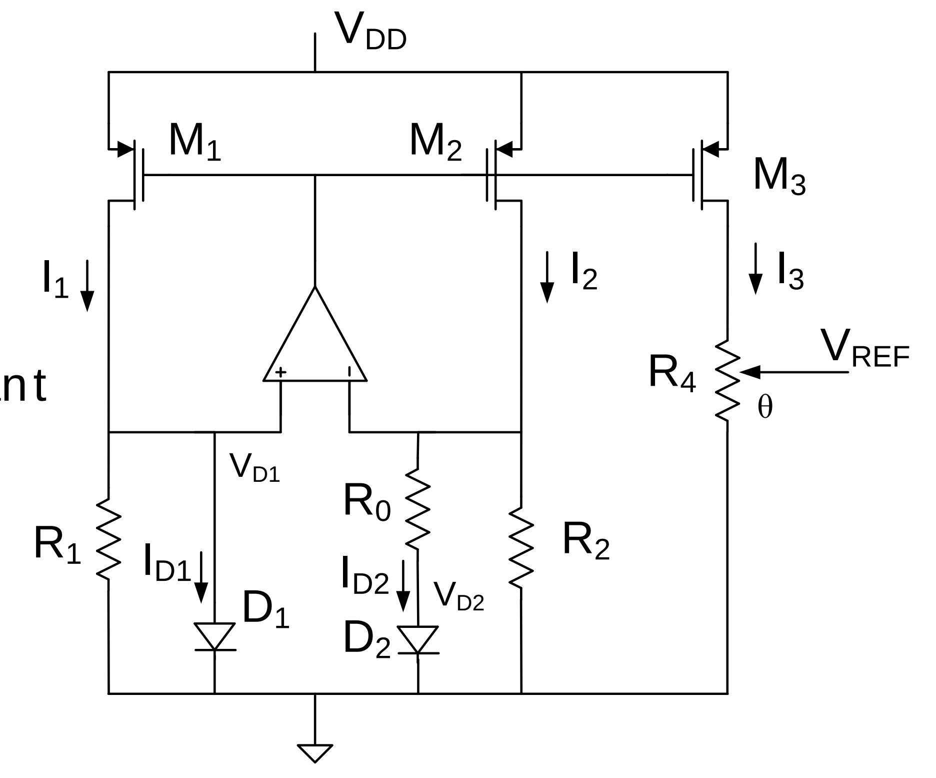
$$I_2 = I_{R0} + I_{R2}$$

$$I_3 = KI_2$$

K is the ratio of I_3 to I_2

$$V_{REF} = \theta I_3 R_4$$

$$\frac{I_{D2}}{I_{D1}} = \text{constant}$$



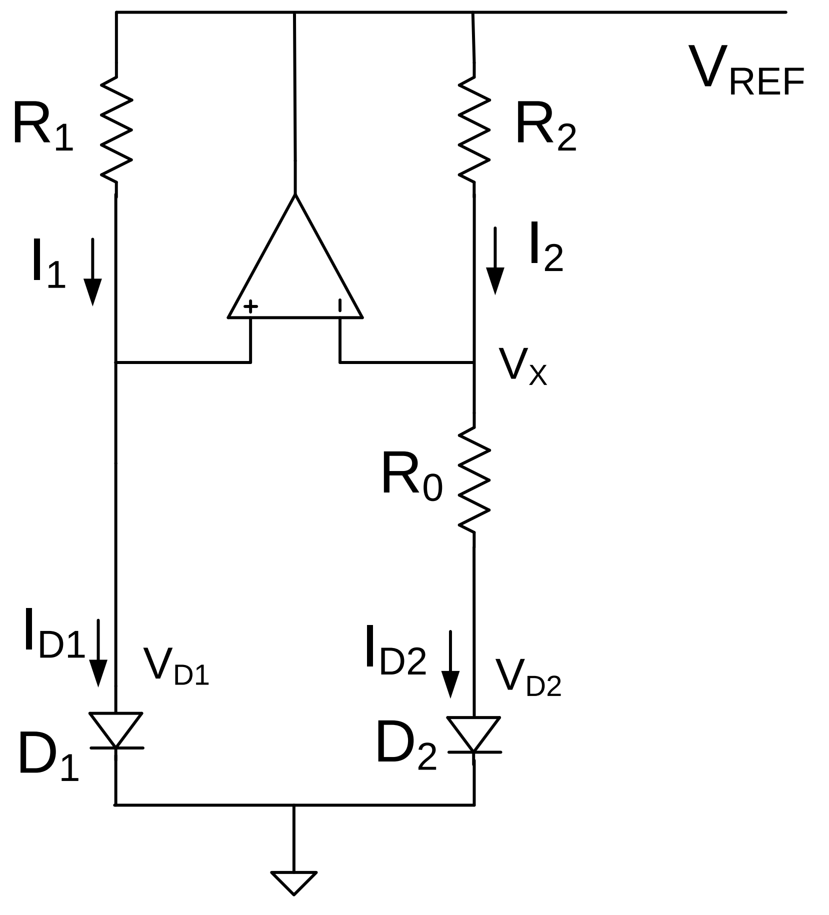
Substituting, we obtain

$$V_{REF} = \theta K R_4 \left(\frac{V_{BE}}{R_1} + \frac{\Delta V_{BE}}{R_0} \right) \rightarrow V_{REF} = \theta K \frac{R_4}{R_1} \left(V_{BE} + \frac{R_1}{R_0} \Delta V_{BE} \right)$$

$$V_{REF} = a_{11} + b_{11} T + c_{11} T \ln T$$

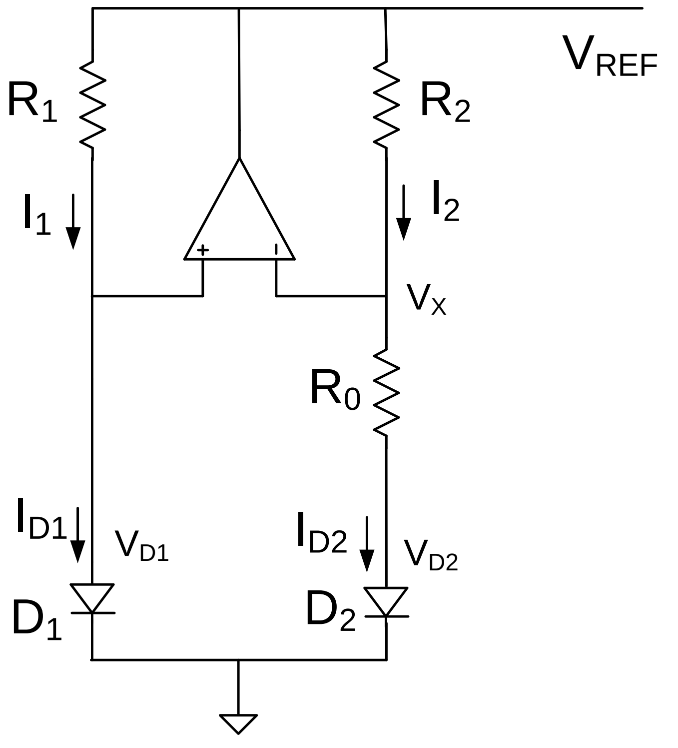
Kujik Bandgap Reference

$$\frac{I_{D2}}{I_{D1}} = constant$$



[12] K. Kuijk, "A Precision Reference Voltage Source", IEEE Journal of Solid State Circuits, Vol. 8, pp. 222-226, June 1973.

Kujik Bandgap Reference



$$I_{R0} = \frac{\Delta V_{BE}}{R_0}$$

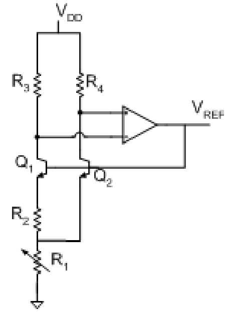
$$I_2 = I_{R0}$$

$$V_{REF} = I_2 R_2 + V_{BE1}$$

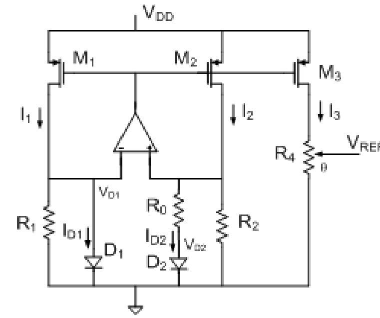
solving, we obtain

$$V_{REF} = \frac{R_2}{R_0} \Delta V_{BE} + V_{BE1}$$

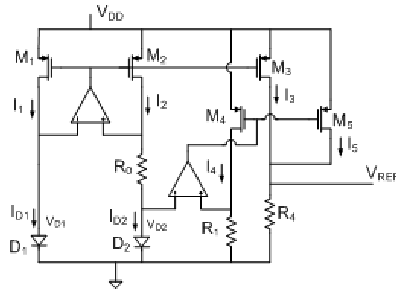
$$V_{REF} = a_{22} + b_{22} T + c_{22} T \ln T$$



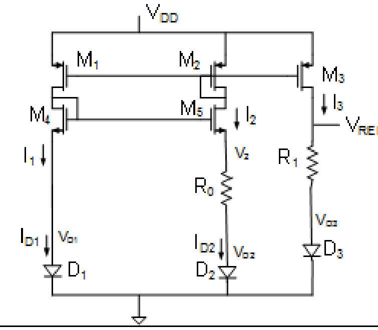
Brokow



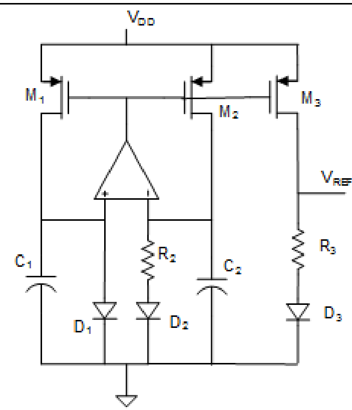
Banba



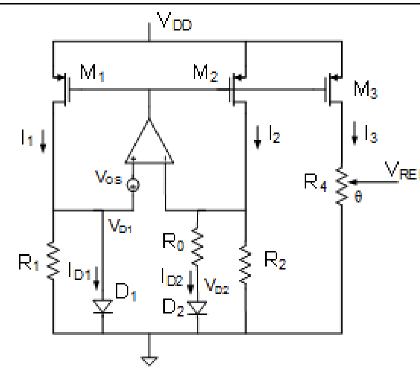
Mietus



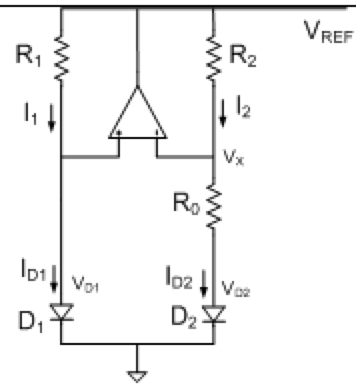
Modified Banba



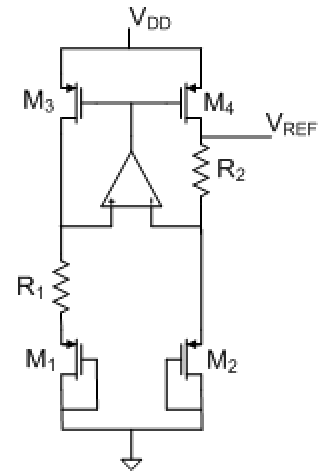
Modified Mietus



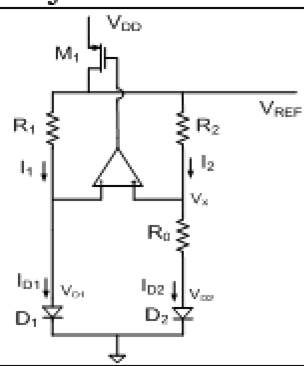
Modified Banba



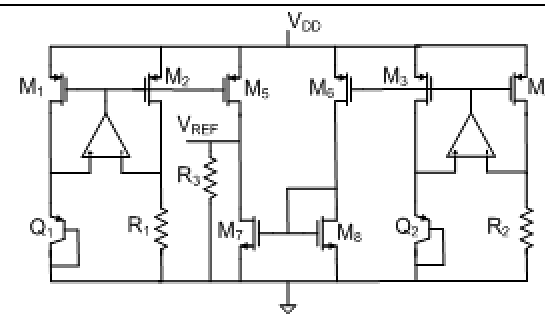
Kuijk



Amema



Modified Kuijk



Zhu

Almost all of the published bandgap references have an output of the form:

$$V_{REF} = a + bT + cT\ln T$$

	a	b	c
Brokow	$a_1 = V_{G0}$	$b_1 = \frac{k}{q}\left(\frac{R_1}{R_2}\left(1+\frac{R_3\alpha_1}{R_4\alpha_2}\right)\ln\left(\frac{R_3}{R_4}\frac{A_{S1}}{A_{S2}}\right)+\ln\left(\frac{k}{q}\frac{R_3}{R_4}\alpha_1\frac{\ln\left(\frac{R_3}{R_1}\frac{A_{S1}}{A_{S2}}\right)}{\tilde{J}_{SX2}R_2}\right)\right)$	$c_1 = \frac{k}{q}(1-m)$
Banba	$a_2 = \left[\frac{R_4}{R_1}\theta K_3\right]V_{G0}$	$b_2 = \left[\frac{k}{q}\theta K_3\right]\left(\frac{R_4}{R_0}\ln\left(\frac{A_{D2}}{A_{D1}}\right)+\frac{R_4}{R_1}\ln\left(\frac{k}{q}\frac{\ln\left(\frac{A_{D2}}{A_{D1}}\right)}{R_0 A_{D1}\tilde{J}_{SX1}}\right)\right)$	$c_2 = \left[\frac{R_4}{R_1}\theta K_3\right]\frac{k}{q}(1-m)$
Mieteus	$a_3 = K_5 V_{G0}$	$b_3 = \frac{k}{q}\left(K_3\frac{R_4}{R_0}\ln\left(K_1\frac{A_{D2}}{A_{D1}}\right)+K_5\left(\ln\frac{k}{q}+\frac{\ln\left(K_1\frac{A_{D2}}{A_{D1}}\right)}{J_{SX}A_{D2}}\right)\right)$	$c_3 = \frac{k}{q}K_5(1-m)$
Kujik	$a_4 = V_{G0}$	$b_4 = \frac{k}{q}\left[\frac{R_2}{R_0}\ln\left(\frac{R_2}{R_1}\frac{A_{D2}}{A_{D1}}\right)+\ln\left(\frac{R_2}{R_1}\frac{k}{q}\frac{\ln\left(\frac{R_2}{R_1}\frac{A_{D2}}{A_{D1}}\right)}{R_0 A_{D1}\tilde{J}_{SX}}\right)\right]$	$c_4 = \frac{k}{q}(1-m)$
Modified Kuijk	$a_5 = V_{G0}$	$b_5 = \frac{k}{q}\left[\frac{R_2}{R_0}\ln\left(\frac{R_2}{R_1}\frac{A_{D2}}{A_{D1}}\right)+\ln\left(\frac{R_2}{R_1}\frac{k}{q}\frac{\ln\left(\frac{R_2}{R_1}\frac{A_{D2}}{A_{D1}}\right)}{R_0 A_{D1}\tilde{J}_{SX}}\right)\right]$	$c_5 = \frac{k}{q}(1-m)$
Modified Kuijk	$a_6 = KV_{G0}$	$b_6 = \frac{k}{q}K\left[\frac{R_2}{R_0}\ln\left(\frac{R_2}{R_1}\frac{A_{D2}}{A_{D1}}\right)+\ln\left(\frac{R_2}{R_1}\frac{k}{q}\frac{\ln\left(\frac{R_2}{R_1}\frac{A_{D2}}{A_{D1}}\right)}{R_0 A_{D1}\tilde{J}_{SX}}\right)\right]$	$c_6 = \frac{k}{q}K(1-m)$
Doyle	$a_6 = V_{G0}$	$b_6 = \frac{k}{q}\left[\frac{K_3}{R_5}\frac{R_1R_3}{R_1+R_2+R_3}\ln\left(K_1\frac{A_{D2}}{A_{D1}}\right)+\frac{R_2+R_3}{R_1+R_2+R_3}\ln\left(\frac{K_4}{R_5}\frac{k}{q}\ln\left(K_1\frac{A_{D2}}{A_{D1}}\right)\right)-\ln(J_{SX}A_{D1})\right]$	$c_6 = \frac{k}{q}\left(\frac{R_2+R_3}{R_1+R_2+R_3}-m\right)$

$$V_{REF} = a + bT + cT \ln T$$

- Start-up Circuits Required on all Bandgap References discussed here
- Bandgap circuits widely used to build voltage references for over 4 decades
- Basic bandgap circuits still used today
- Trimming often required to set inflection point at desired temperature
- Offset voltage of Op Amp and TCR of resistors degrade performance
- Experimental performance often a factor of 2 to 10 worse than that predicted here but still quite good
- Ongoing research activities focusing on improving performance of bandgap references

Stay Safe and Stay Healthy !

End of Lecture 39